

**A framework for embedding
molecular-level information in
continuum-scale simulations of
interfacial flows**

APS 2015

11:01 AM–11:14 AM

By

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In collaboration with Panagiotis Theodorakis,
Erich Muller, Richard Craster and Omar Matar

Computational Fluid Dynamics (CFD)

1) G. Karapetsas, R. Craster
& O. Matar, *JFM*, 2011

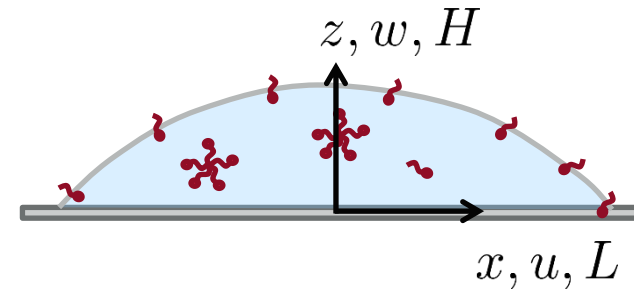
- Incompressible Navier Stokes with the thin-film approximation.

$$\frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial z^2} \quad \frac{\partial P}{\partial z} = 0 \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

- With boundary conditions

$$P = - \left(\frac{H}{L} \right)^2 \frac{\partial^2 h}{\partial x^2} \left(\sigma_l + \frac{1}{\Sigma_l} \right) \quad \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = w \quad \frac{\partial u}{\partial z} = \frac{\partial \sigma_l}{\partial x} \quad z = h$$

$$u = \beta \frac{\partial u}{\partial z} \quad w = 0 \quad z = 0$$

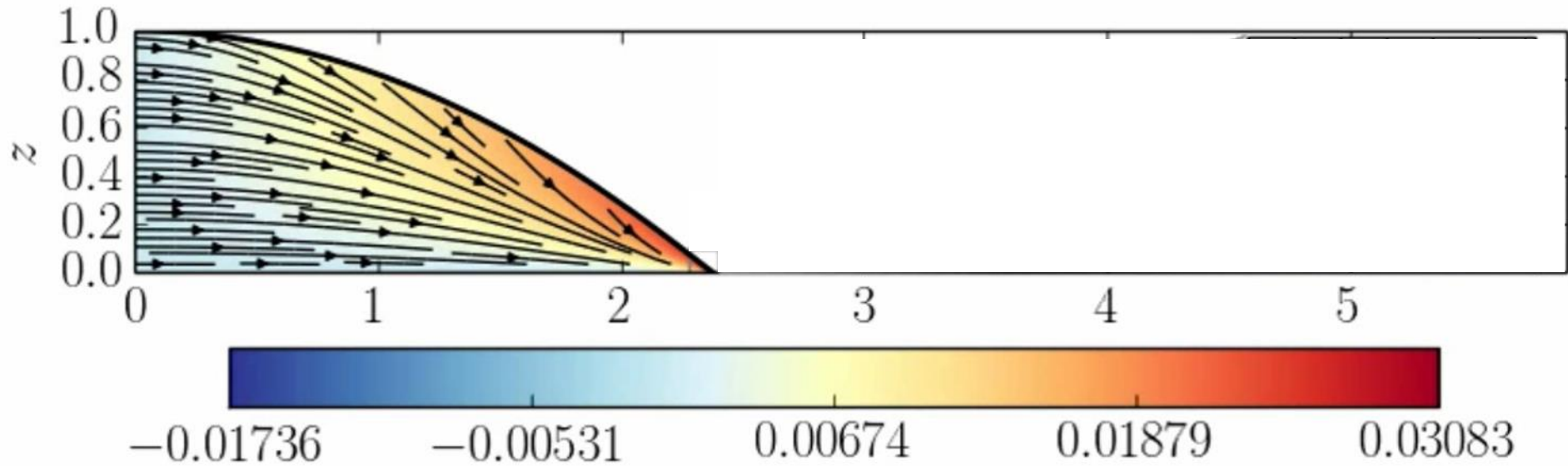


- Surfactant** modelled by advection-diffusion equations with empirical sorption processes - **coupled to the dynamics through surface tension**
- Contact line evolution is modelled by an empirical law

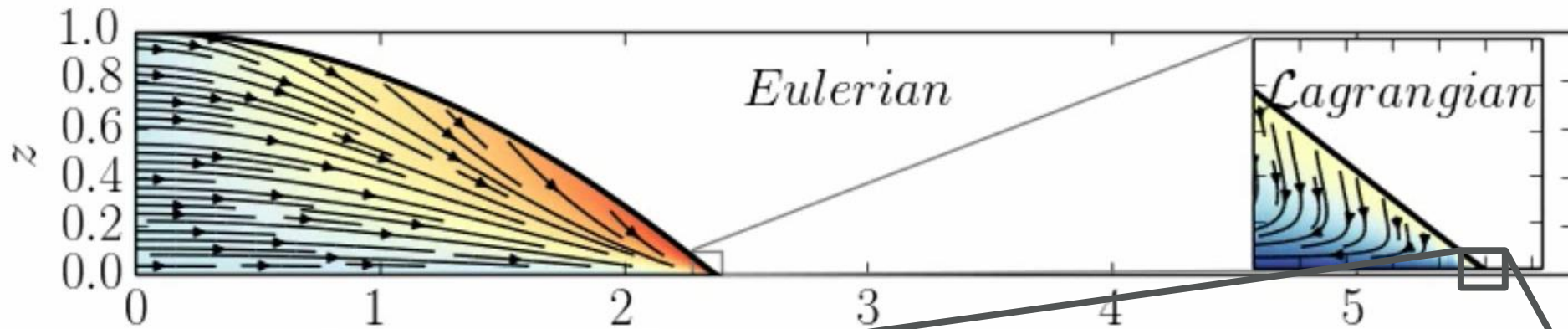
$$\frac{dx_c}{dt} = k(\theta - \theta_a)^n$$

- The angle coupled to **surfactant absorption** at the contact line is essential⁽¹⁾

Coupled Droplet Spreading and MD

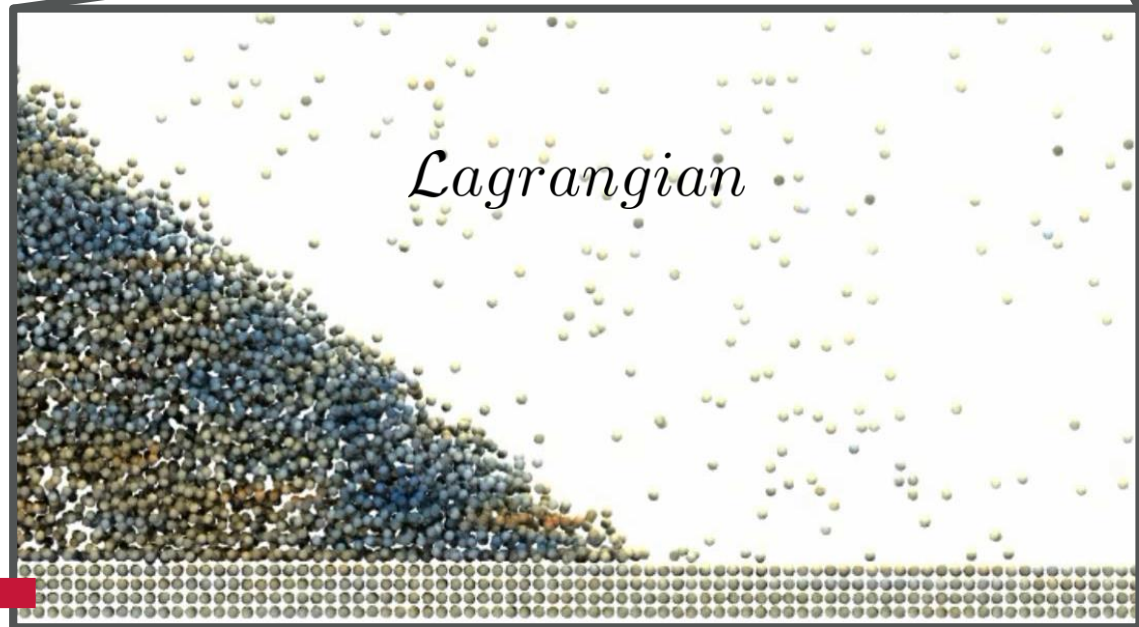


Coupled Droplet Spreading and MD



- Model the moving contact line with MD
- We want contact line speed as a function of continuum contact angle

$$\frac{dx_c}{dt}$$



Molecular Dynamics

Discrete molecules in continuous space

- Molecular position evolves continuously in time
- Position and velocity from acceleration

$$\ddot{\mathbf{r}}_i \rightarrow \dot{\mathbf{r}}_i$$

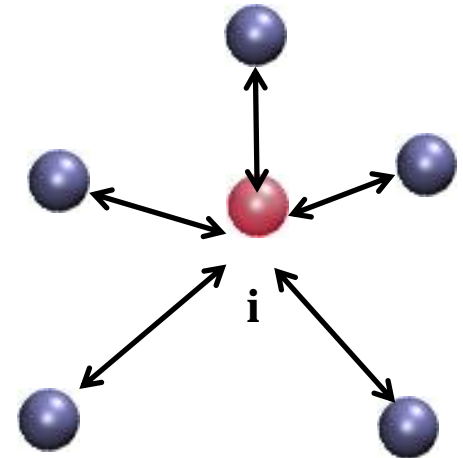
$$\dot{\mathbf{r}}_i \rightarrow \mathbf{r}_i(t)$$

Acceleration obtained from forces

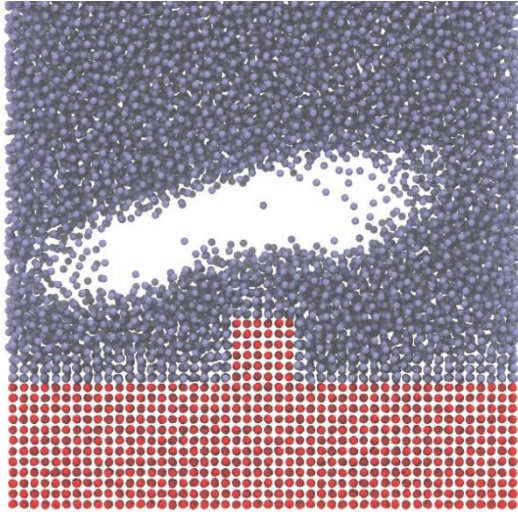
- Governed by Newton's law for an N-body system
- Point particles with pairwise interactions only

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i = \sum_{i \neq j}^N \mathbf{f}_{ij} \quad \Phi(r_{ij}) = 4\epsilon \left[\left(\frac{\ell}{r_{ij}} \right)^{12} - \left(\frac{\ell}{r_{ij}} \right)^6 \right]$$

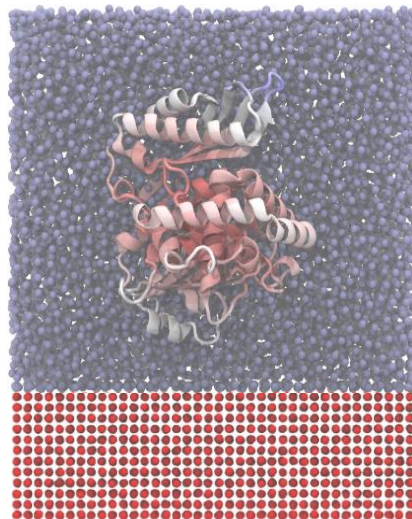
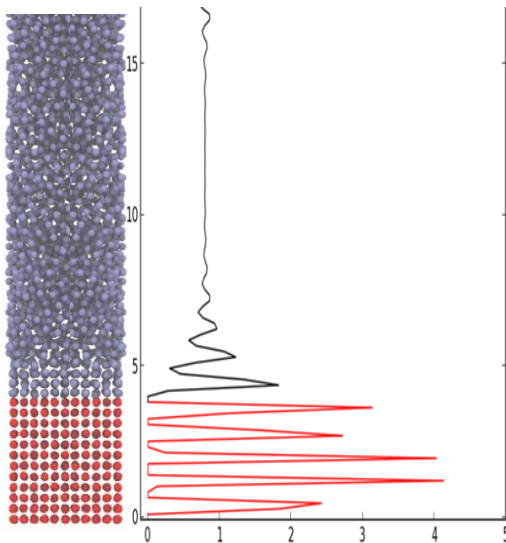
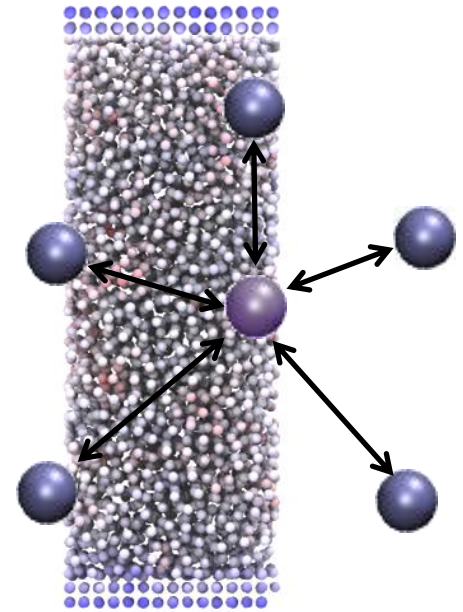
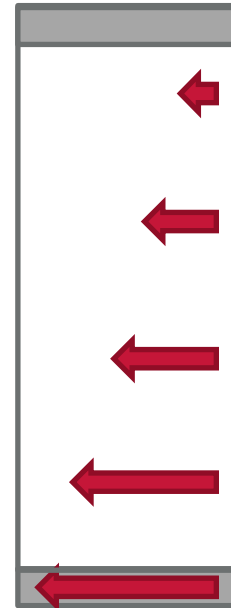
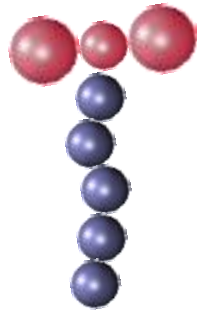
- SAFT¹⁾ using the γ -Mie²⁾ potential $\Phi(r_{ij}) = 4C\epsilon_{ij} \left[\left(\frac{\ell_{ij}}{r_{ij}} \right)^{\lambda_r} - \left(\frac{\ell_{ij}}{r_{ij}} \right)^{\lambda_a} \right]$



Molecular Dynamics



*Superspreading
Surfactant, e.g.
Silwet-L77*



A Better way of Getting the Contact Angle

VOLUME 63, NUMBER 7

PHYSICAL REVIEW LETTERS

14 AUGUST 1989

Simulations of Contact-Line Motion: Slip and the Dynamic Contact Angle

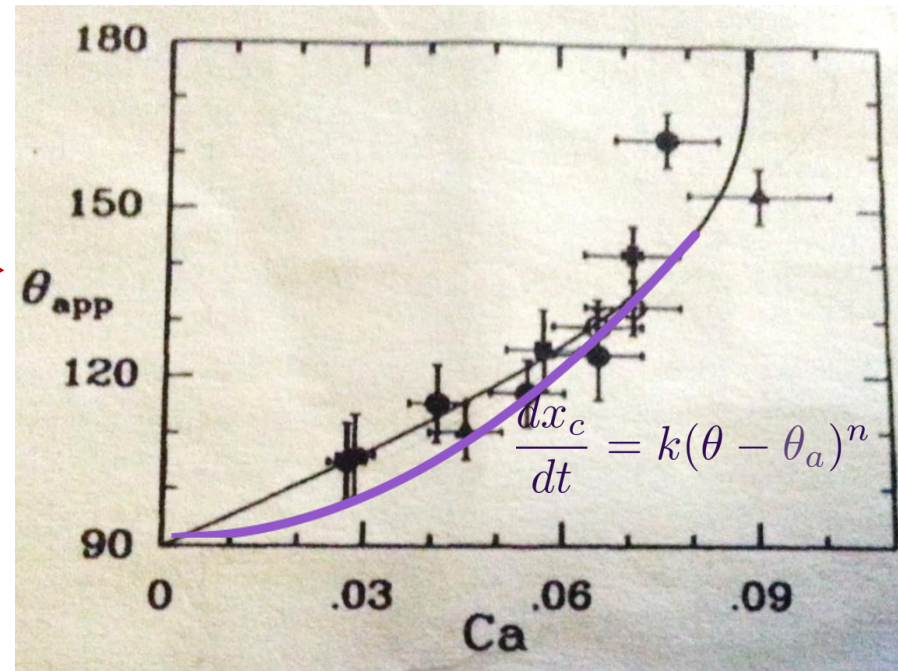
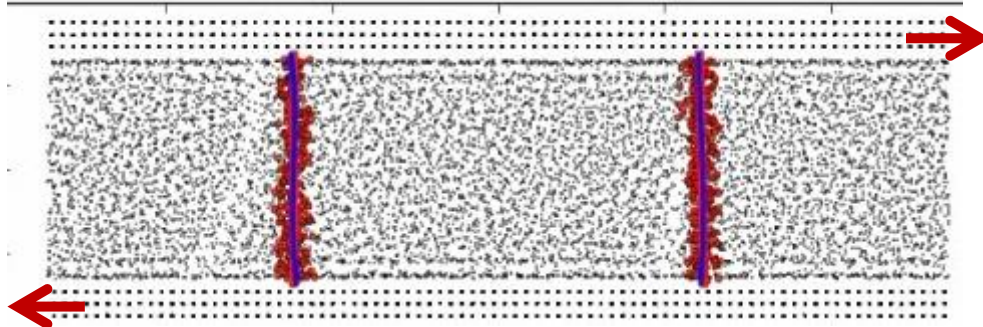
Peter A. Thompson and Mark O. Robbins

Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218

(Received 7 February 1989)

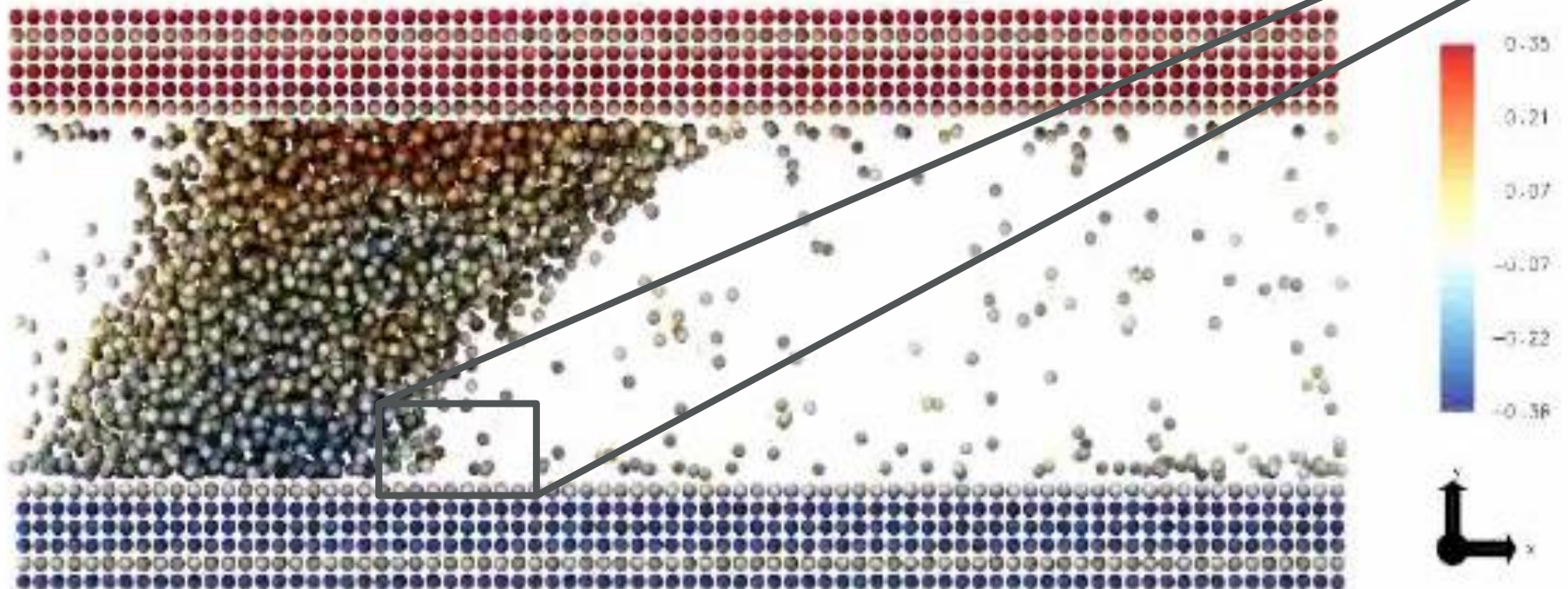
- Reproduces Cox's Law

Bottom angle = 90.10 and top angle = 88.85

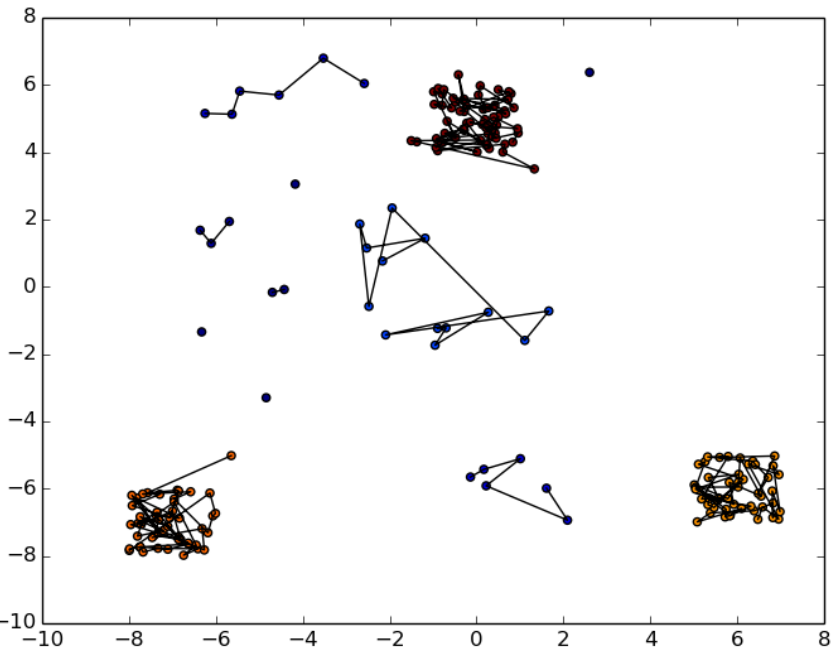


Two phase version closer to experimental reality

- Two fluid phases and sliding molecular walls
- Simple test case to explore wall velocity vs contact line angle
- Non-Equilibrium Steady State

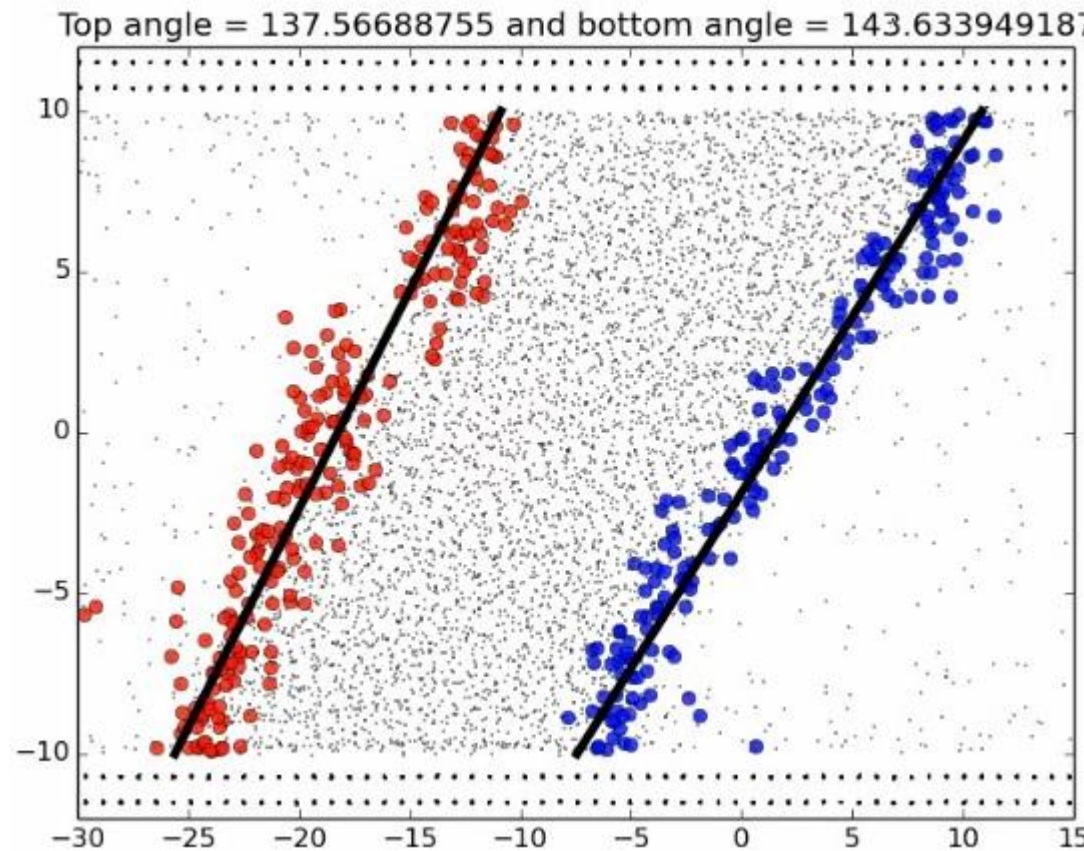


Cluster analysis and surface fitting



- Cluster analysis

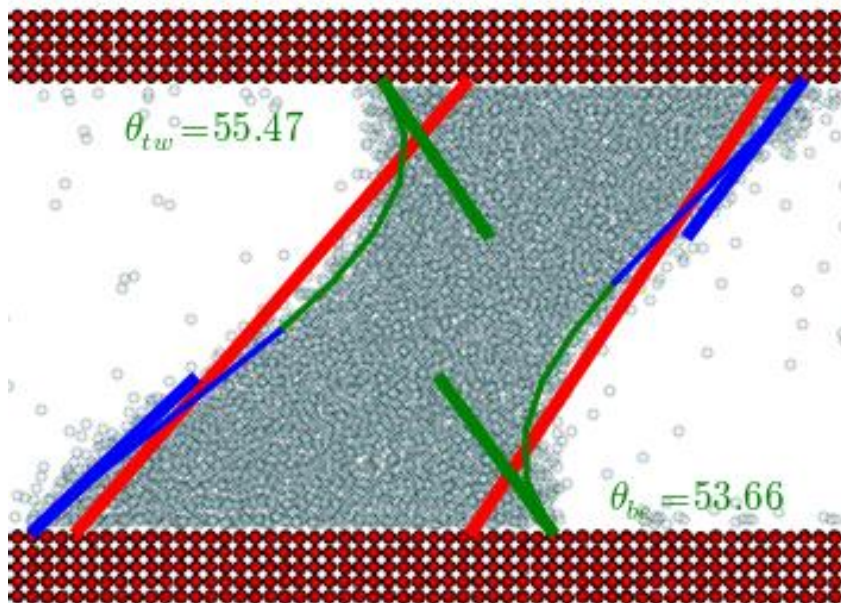
- Finding the fluid-liquid interface



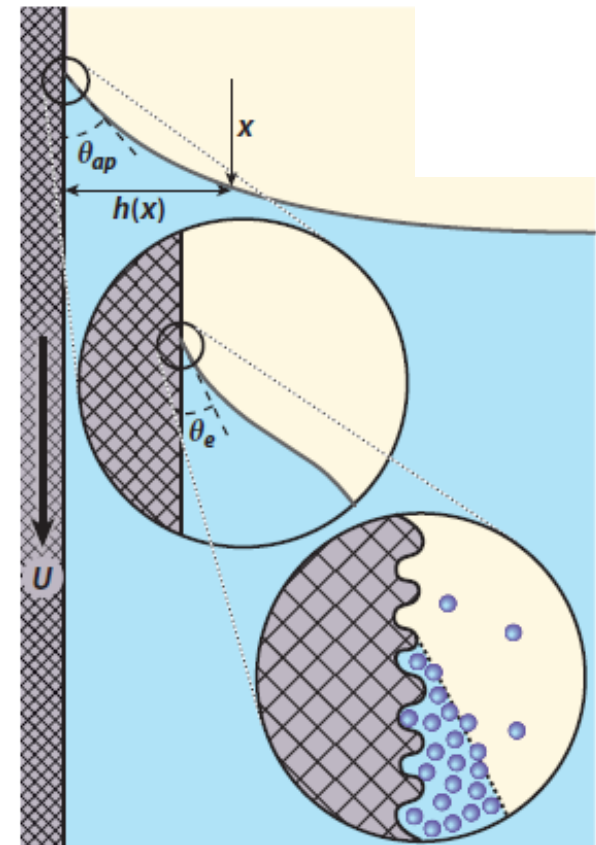
Determining the Contact angle

1) Snoeijer, Andreotti (2013) Annual
Rev Fluid Mech 45:269–92

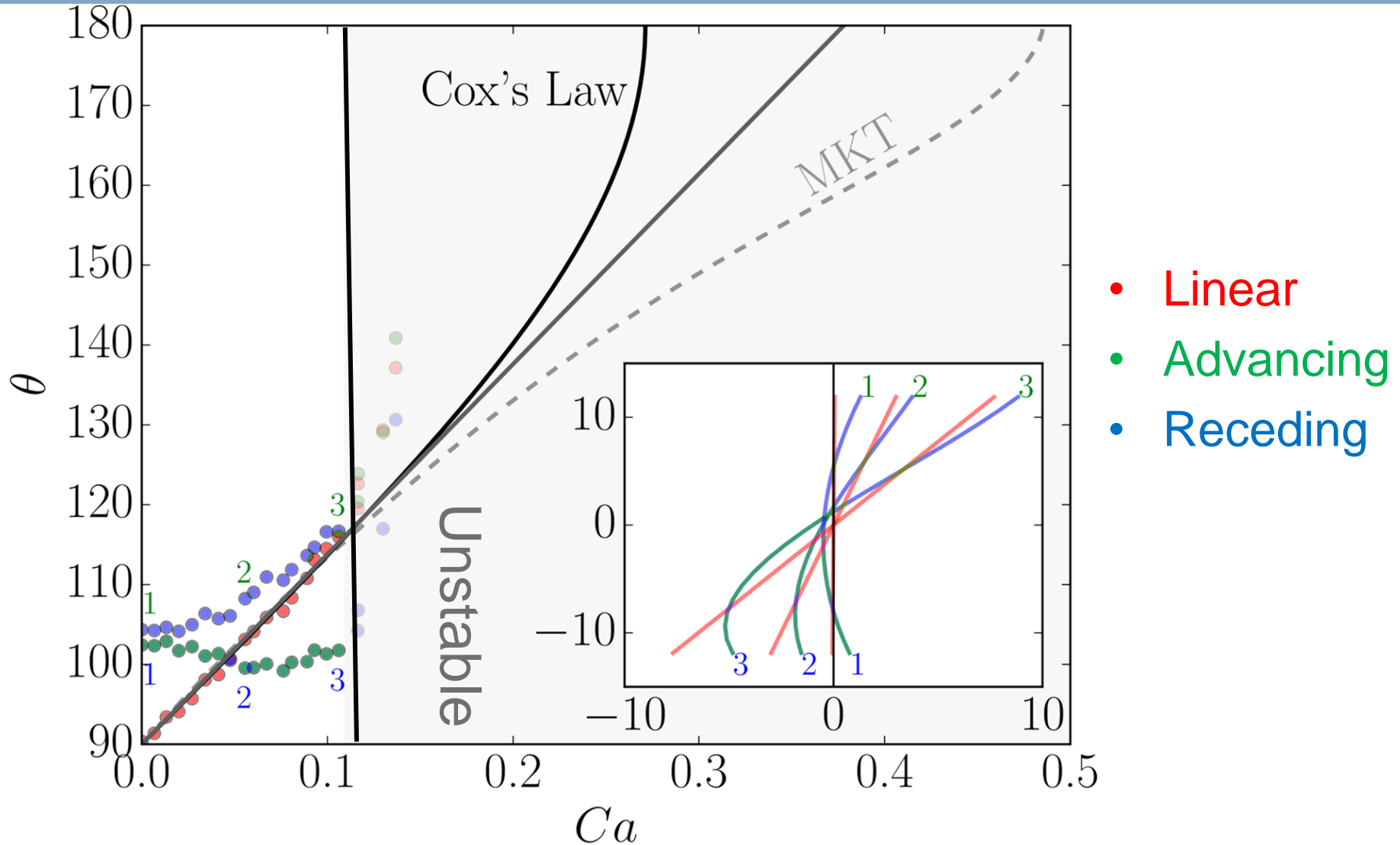
- Linear, Advancing and Receding angles



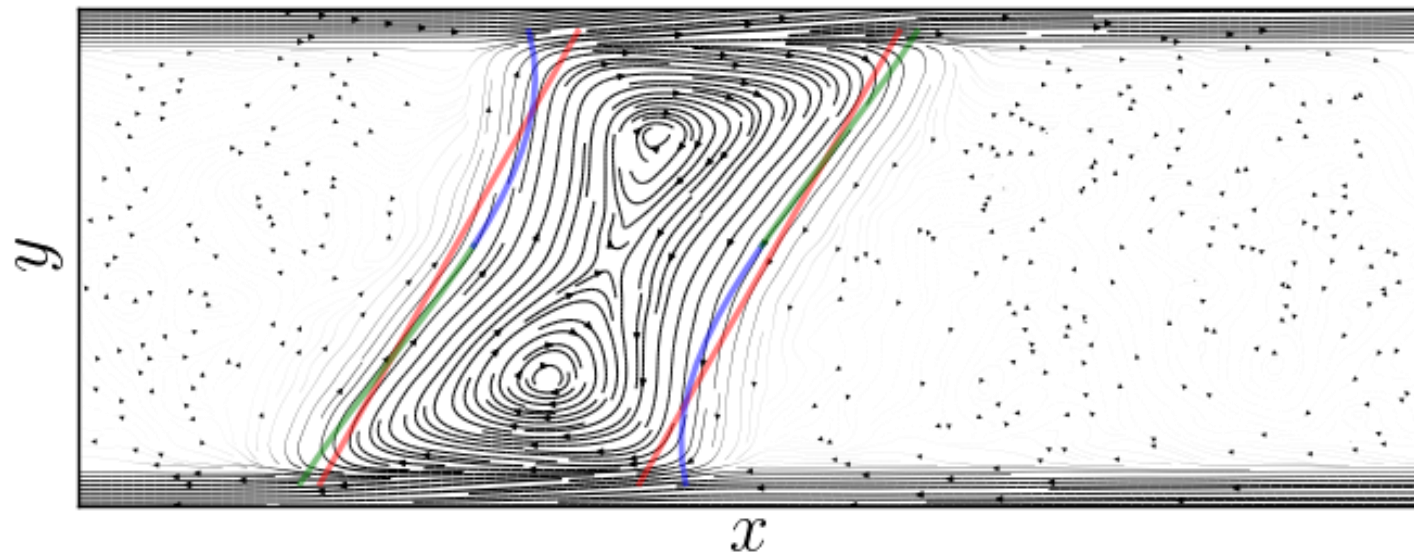
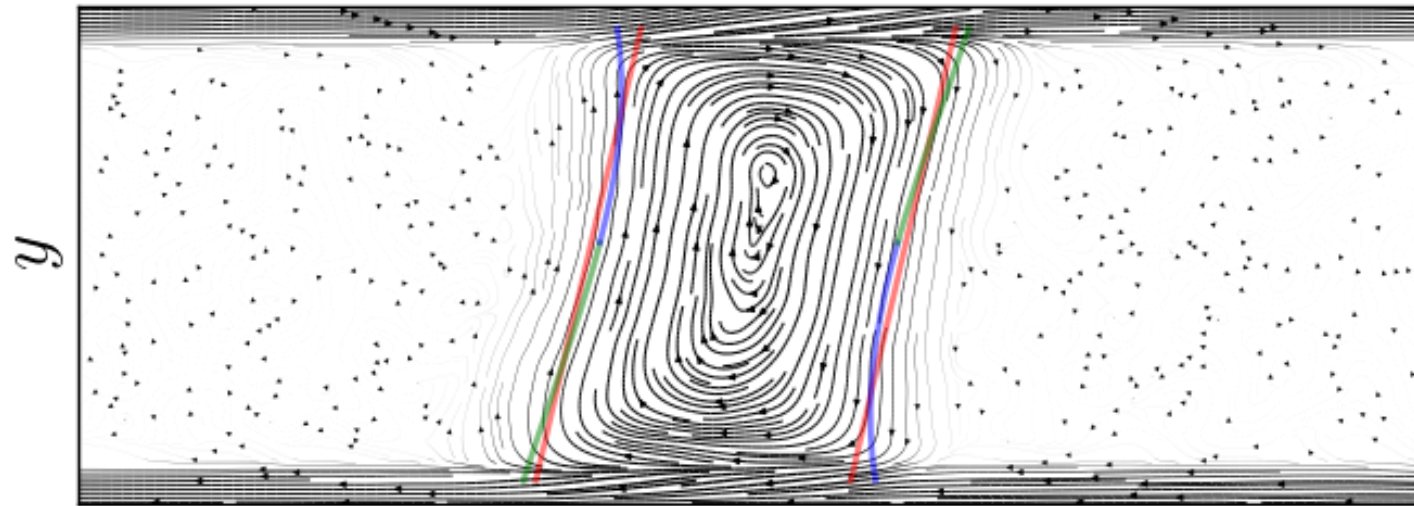
1)



Contact angles vs sliding velocity

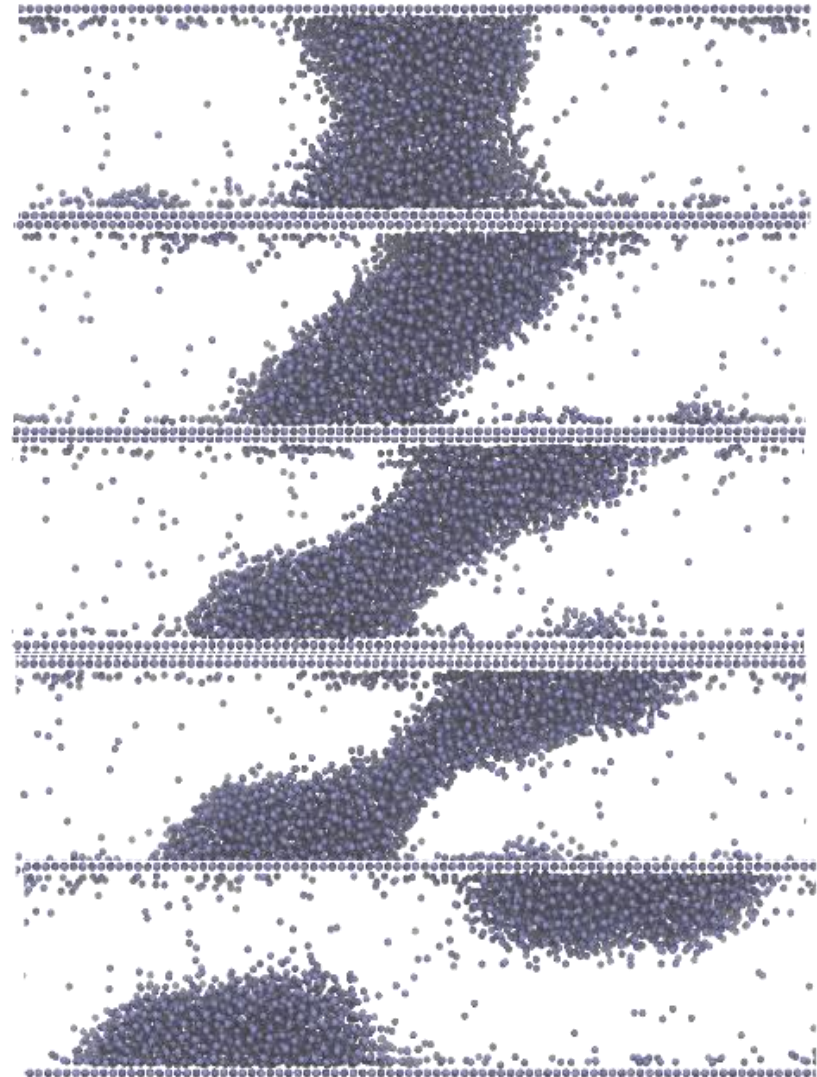
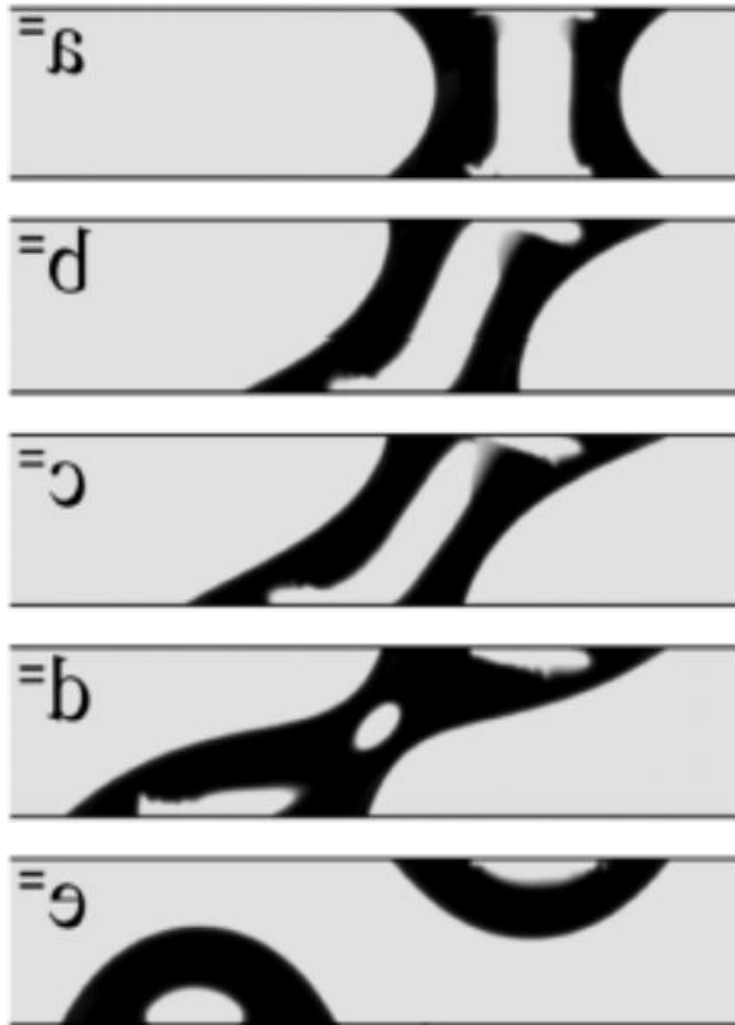


Streamlines



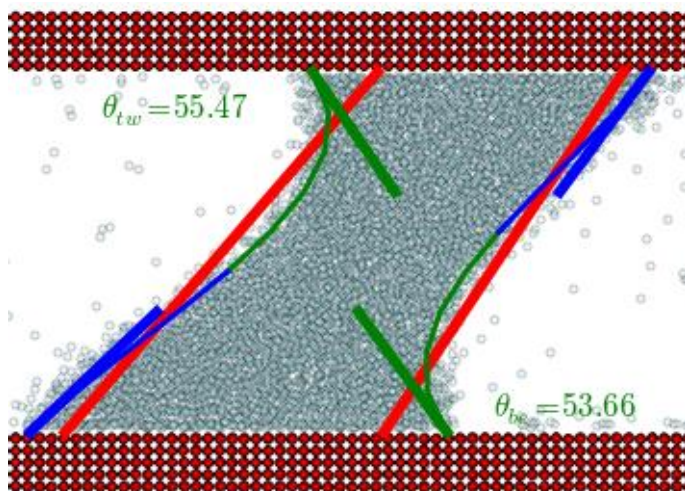
Droplet Breakdown

L. Wang, T. J. McCarthy (2013) Shear Distortion and Failure of Capillary Bridges. Wetting Information Beyond Contact Angle Analysis Langmuir 29, 7776–7781

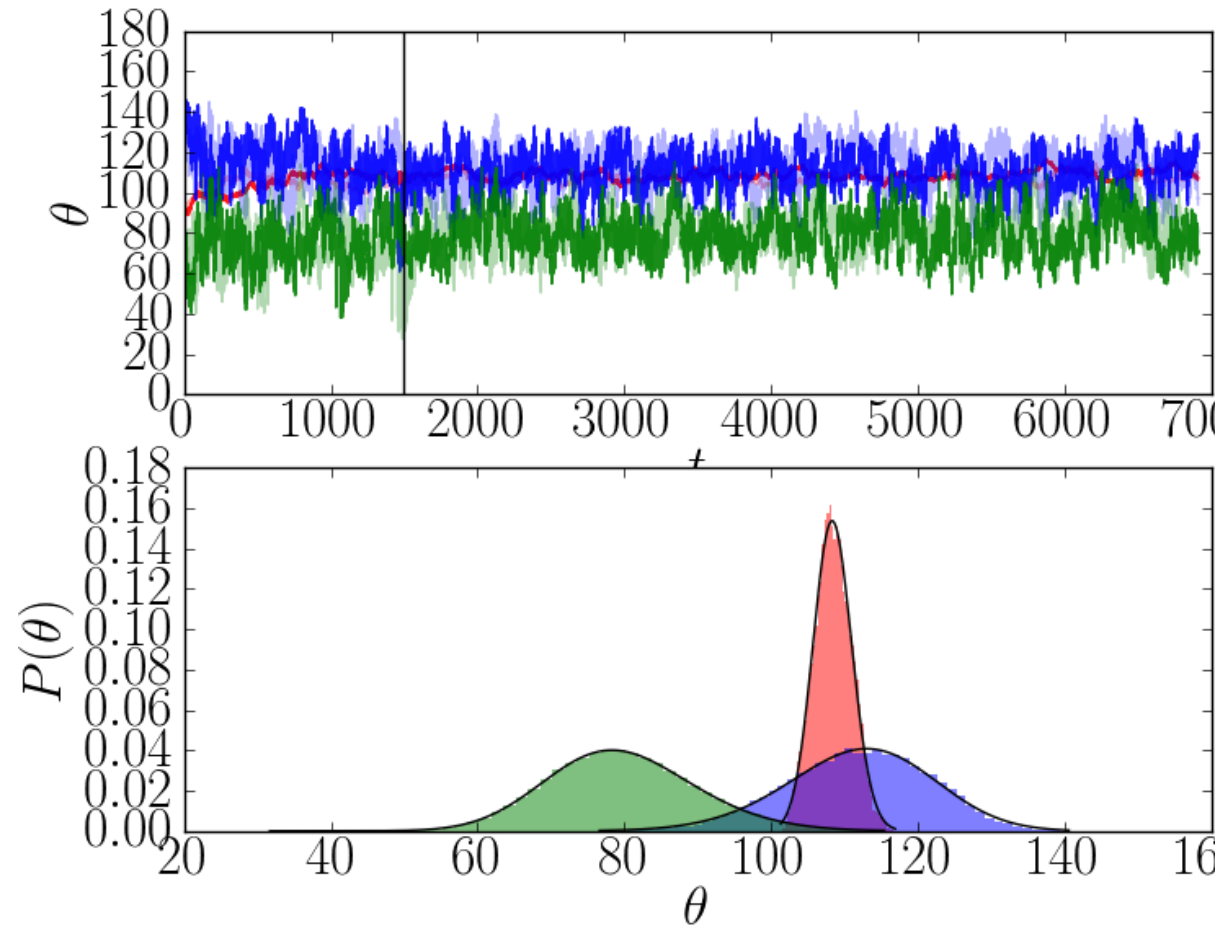


Time Evolution of Contact Angle

- Plot evolution of various contact angles as a function of time

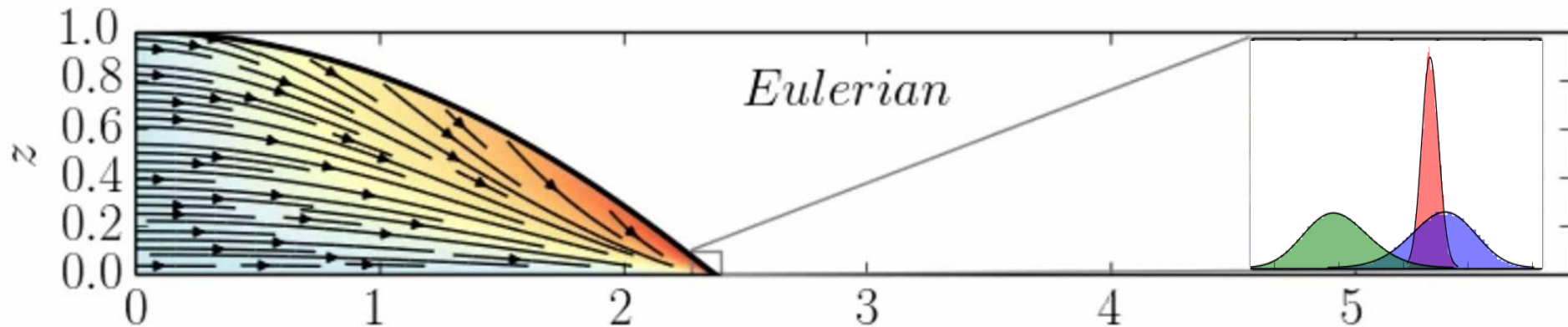


- Probability density function of angle shows range of micro-scale behaviour



- Linear**, **Advancing** and **Receding** angles

Building this into the Continuum Model



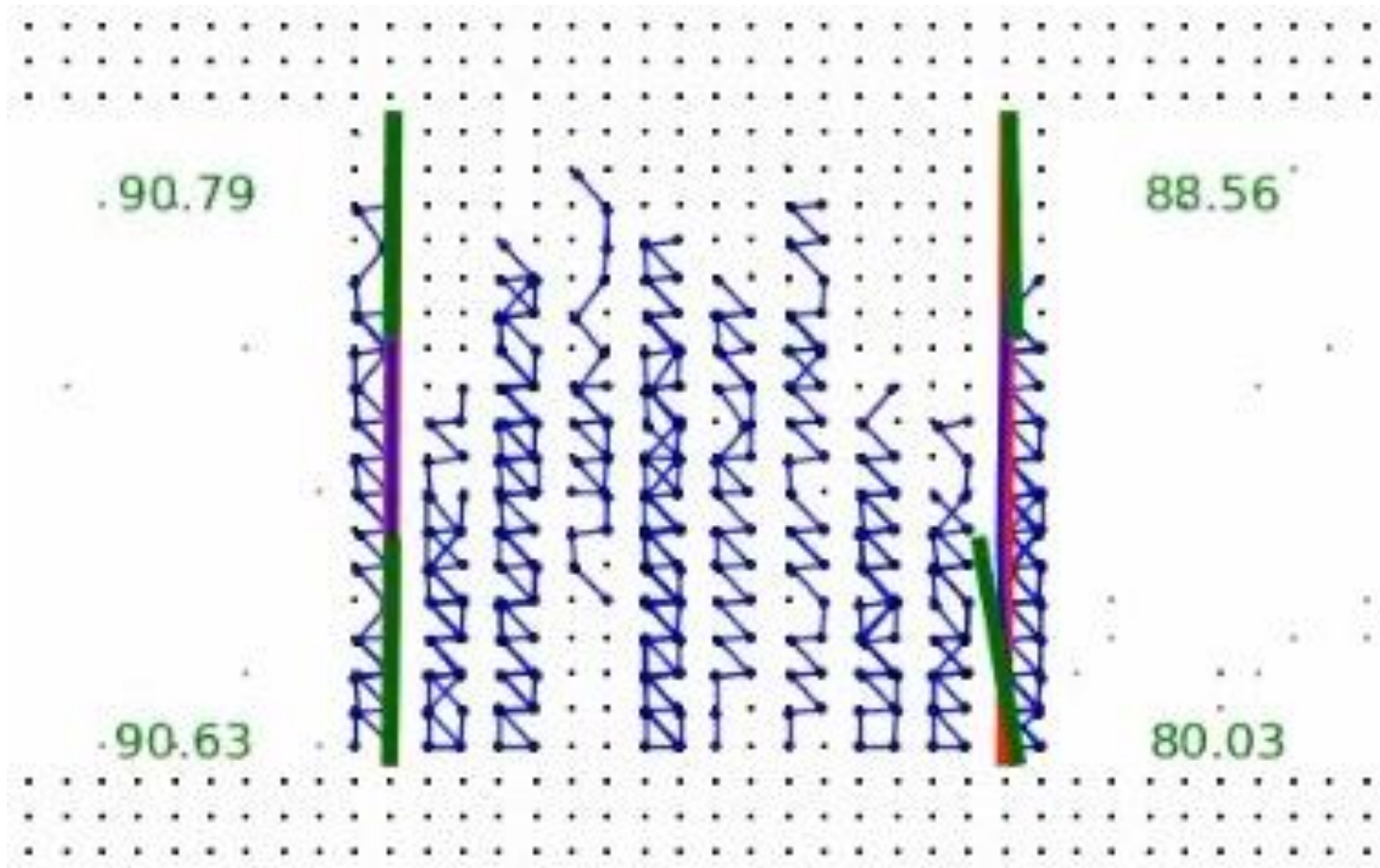
- Speed of contact line can be chosen from the molecular PDF with the appropriate speed in a simple fluid model

$$\frac{dx_c}{dt} = k(\theta - \theta_a)^n$$

- The more complex surfactant case requires simulations to be run dynamically for current surfactant concentration

Surfactant Spreading More Complex

- Surfactant concentration impacts contact angle
 - Includes mechanism for absorption at contact line

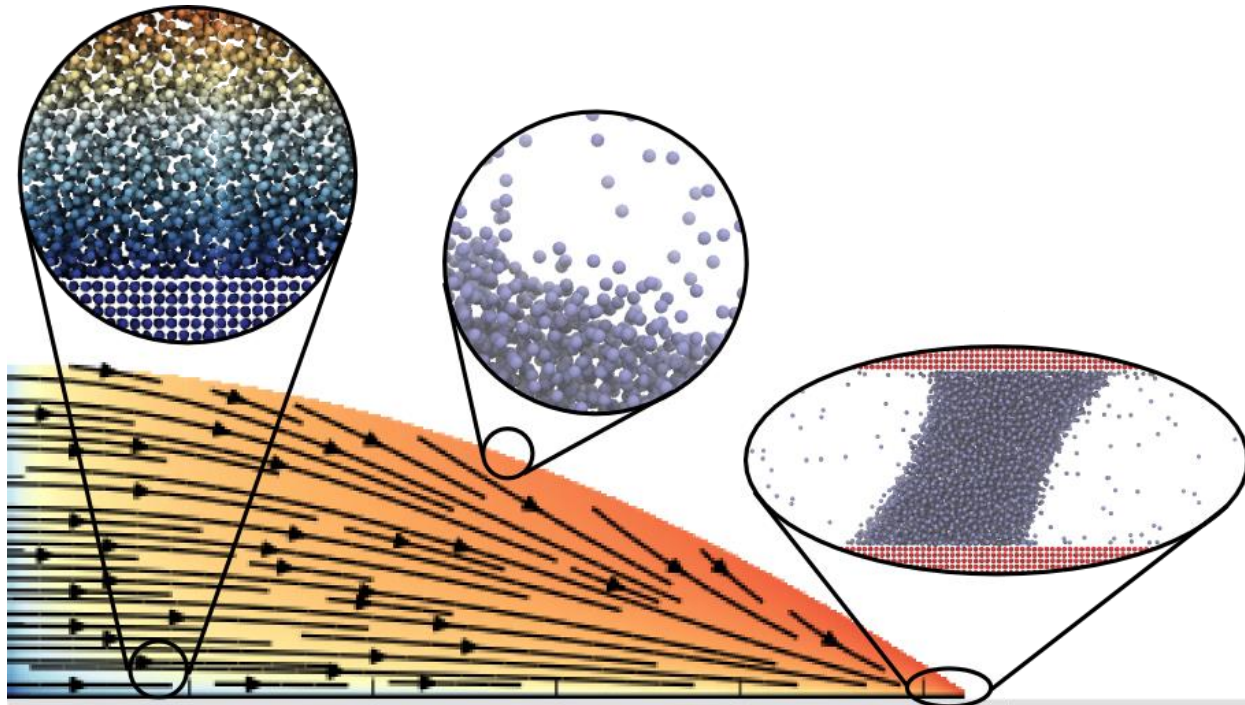


Conclusions and Future Aims

- Molecular dynamics provides a more detailed model of the contact line, multiple phases and wall-fluid interactions
- We want to combine both in a single model
 - For simple cases, probability density functions of molecular detail
 - For surfactant simulations the reference solutions could be run on the fly based on surfactant concentration
 - Longer term domain decomposition using a detailed continuum model
- Choice of methodology depends on phenomenon of interest
 - But any molecular modelling represents an improvement on empirical models.

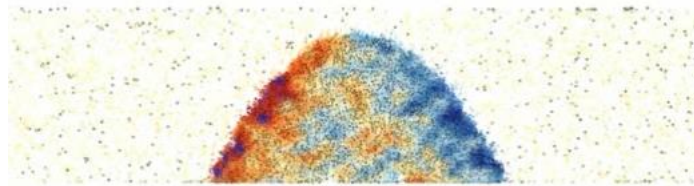
Acknowledgements

- Current work funded by EPSRC Grant number EP/J010502/1
- Working with Erich Muller, Richard Craster and Omar Matar

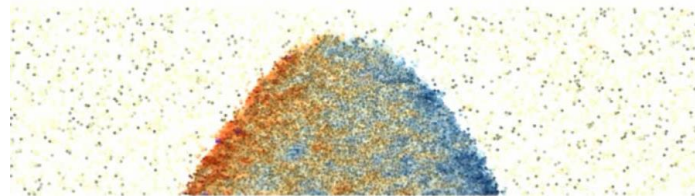


MD Simulation of Droplets

- Low Wettability



- Intermediate Wettability



- High Wettability



Coupling Overview

- 1) Ren (2007), E et al (2003), Borg et al (2013)
- 2) O'Connell and Thompson (1995), Flekkøy et al (2000), Nie et al (2004), Hadjiconstantinou et al (1999), Delgado-Buscalioni and Coveney, (2003)

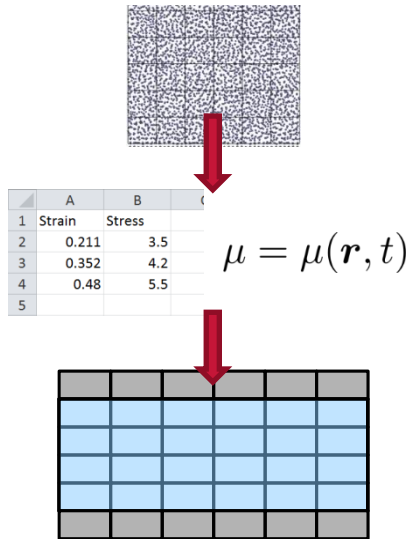
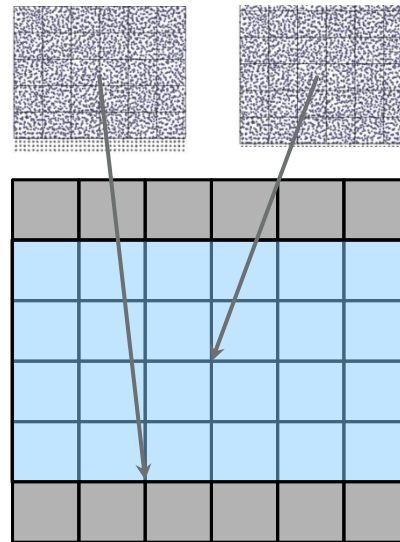


Table Lookup or Coefficients

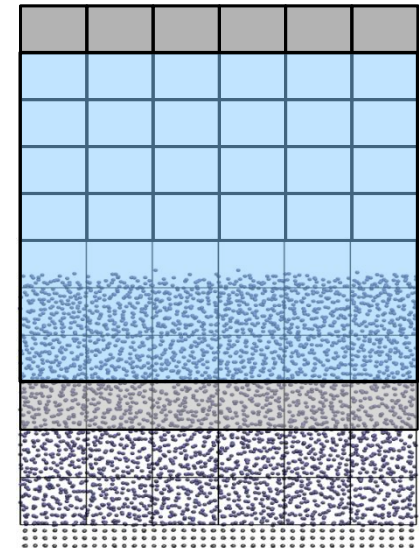
MD parameter study stored in table and CFD uses data



Embedded Models

MD – embedded in a CFD simulation

Used for Non-Newtonian effects ¹⁾



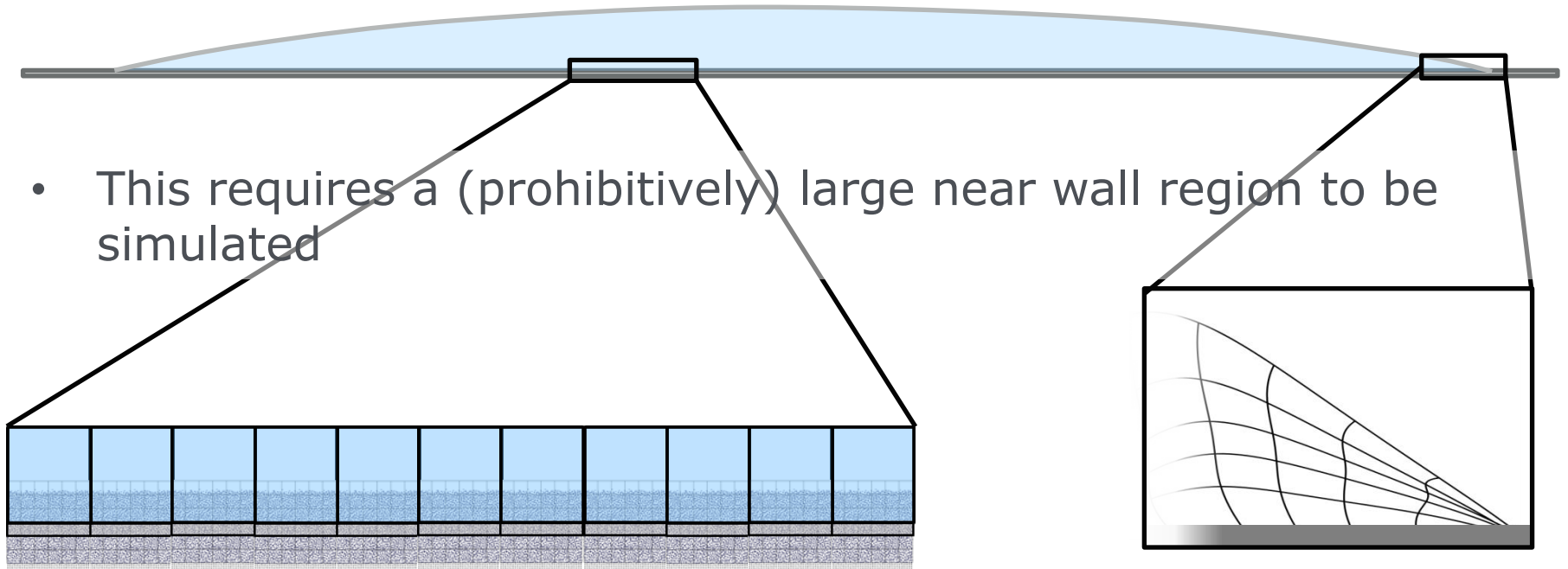
Domain Decomposition

MD –CFD linked along an interface

Local features e.g. contact line ²⁾

Droplet Modelling

- Thin film equations assumes a large length to height ratio



- This requires a (prohibitively) large near wall region to be simulated

- Especially when only the contact line dynamics are of interest
 - Stretched grid and moving interface are complex to model

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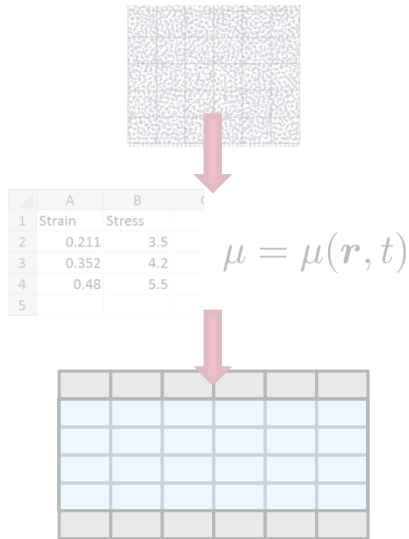
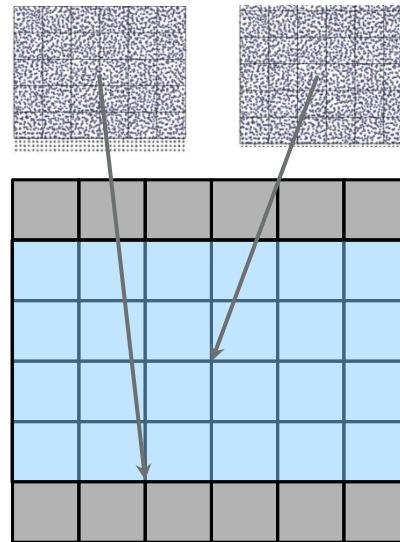


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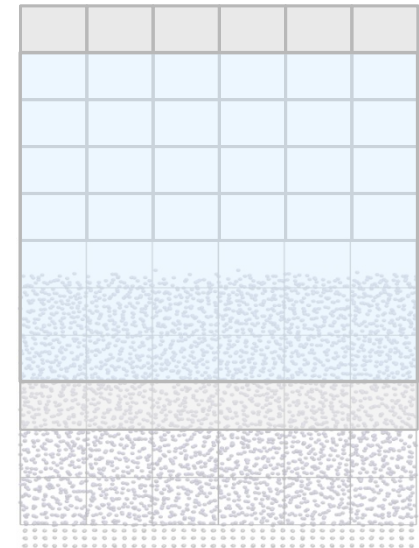
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