

# Multi-Scale Strategies for Dealing with Moving Contact Lines

**APS 2017**  
**11:14AM–11:27AM**

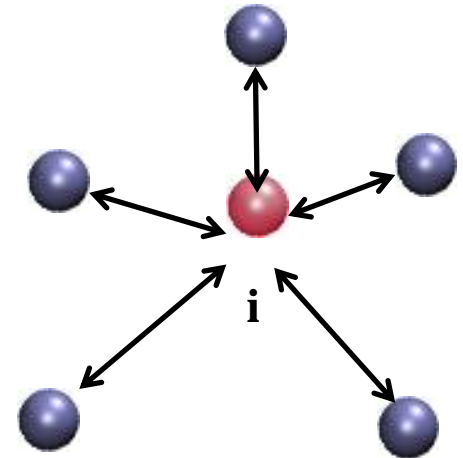
By  
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Panos Theodorakis, Richard Craster  
and Omar Matar

# Molecular Dynamics

## Discrete molecules in continuous space

- Molecular position evolves continuously in time
- Position and velocity from acceleration

$$\begin{aligned}\ddot{\mathbf{r}}_i &\rightarrow \dot{\mathbf{r}}_i \\ \dot{\mathbf{r}}_i &\rightarrow \mathbf{r}_i(t)\end{aligned}$$

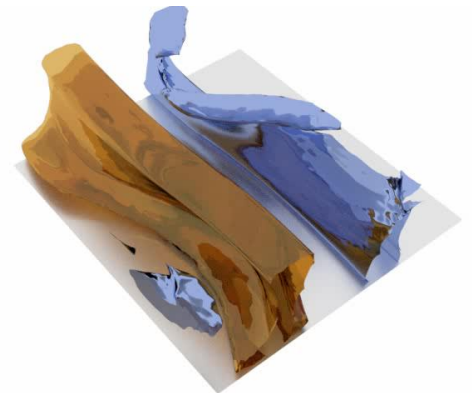
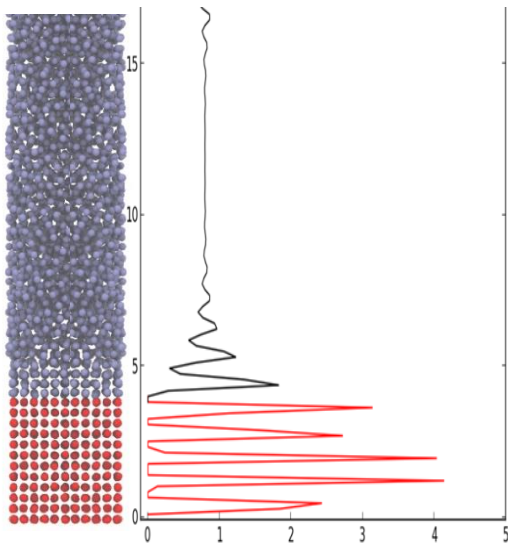
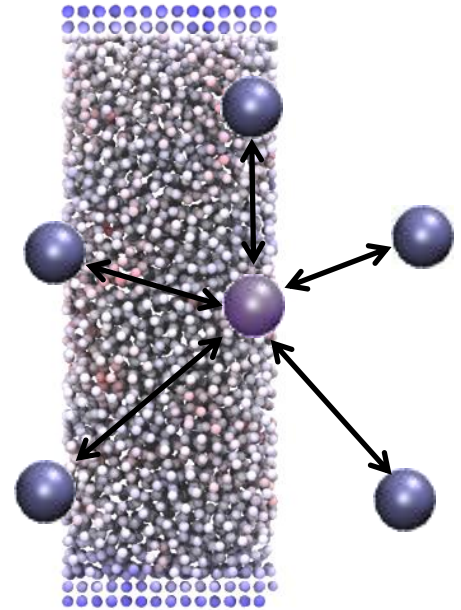
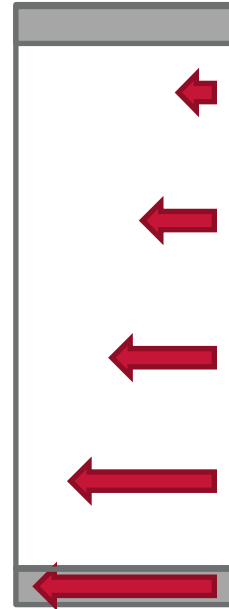
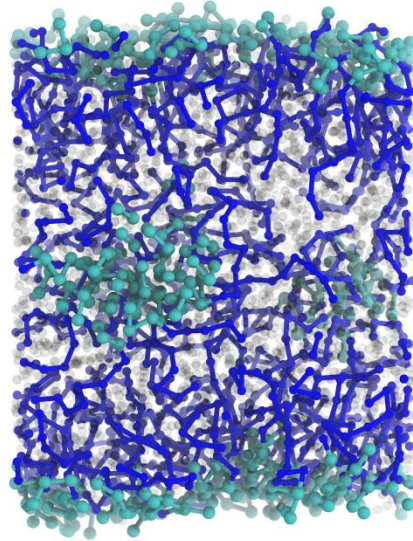


## Acceleration obtained from forces

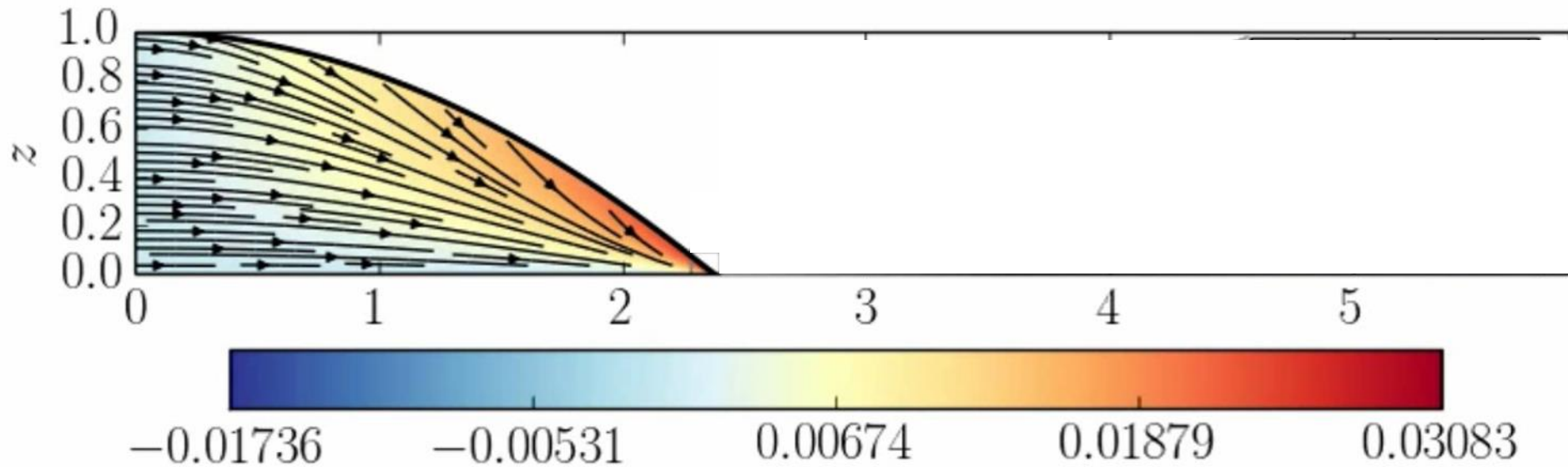
- Governed by Newton's law for an N-body system
- Point particles with pairwise interactions only

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i = \sum_{i \neq j}^N \mathbf{f}_{ij} \quad \Phi(r_{ij}) = 4\epsilon \left[ \left( \frac{\ell}{r_{ij}} \right)^{12} - \left( \frac{\ell}{r_{ij}} \right)^6 \right]$$

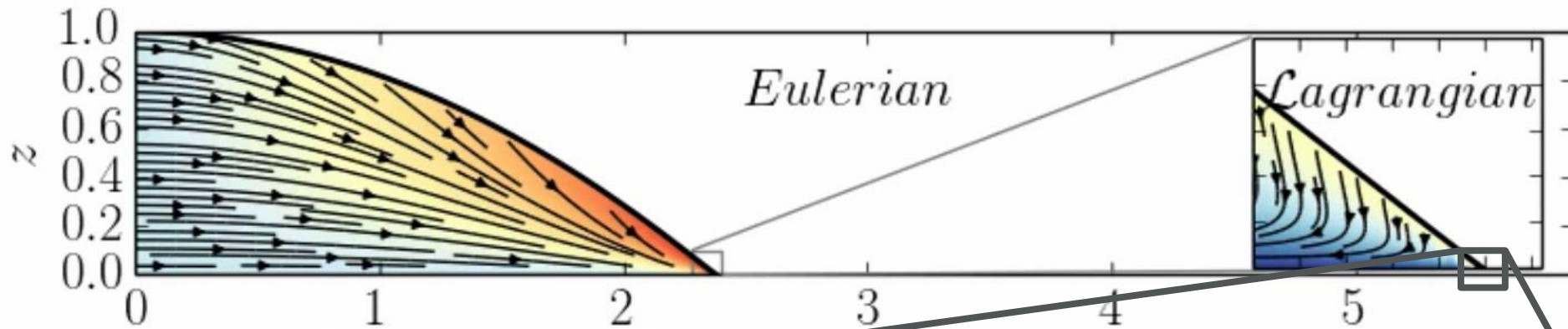
# Molecular Dynamics



# Coupled Droplet Spreading and MD

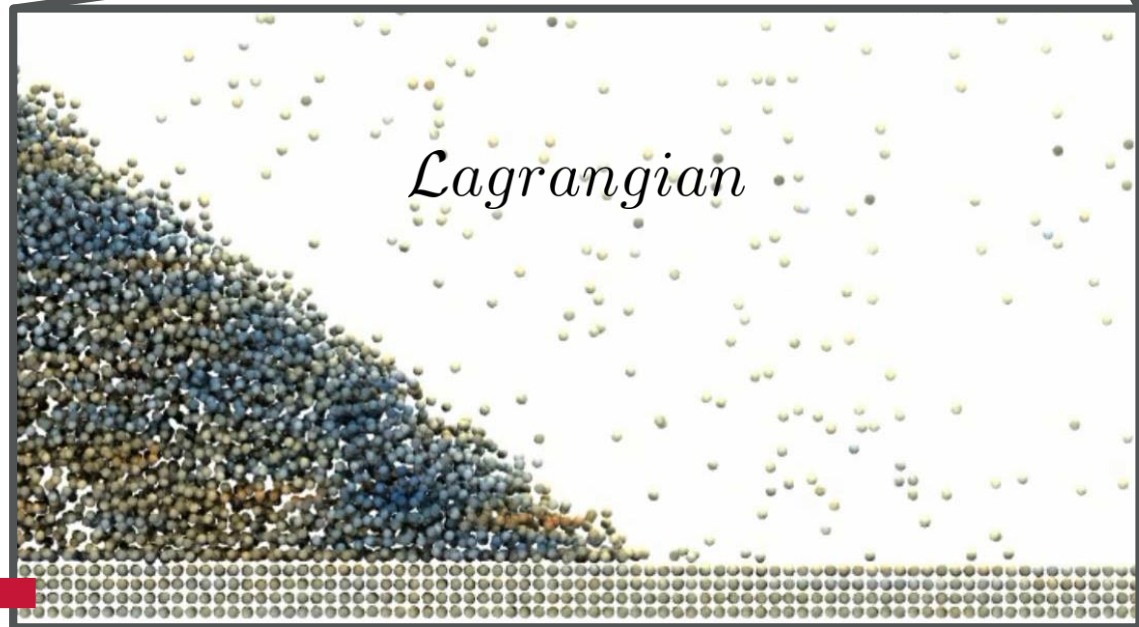


# Coupled Droplet Spreading and MD



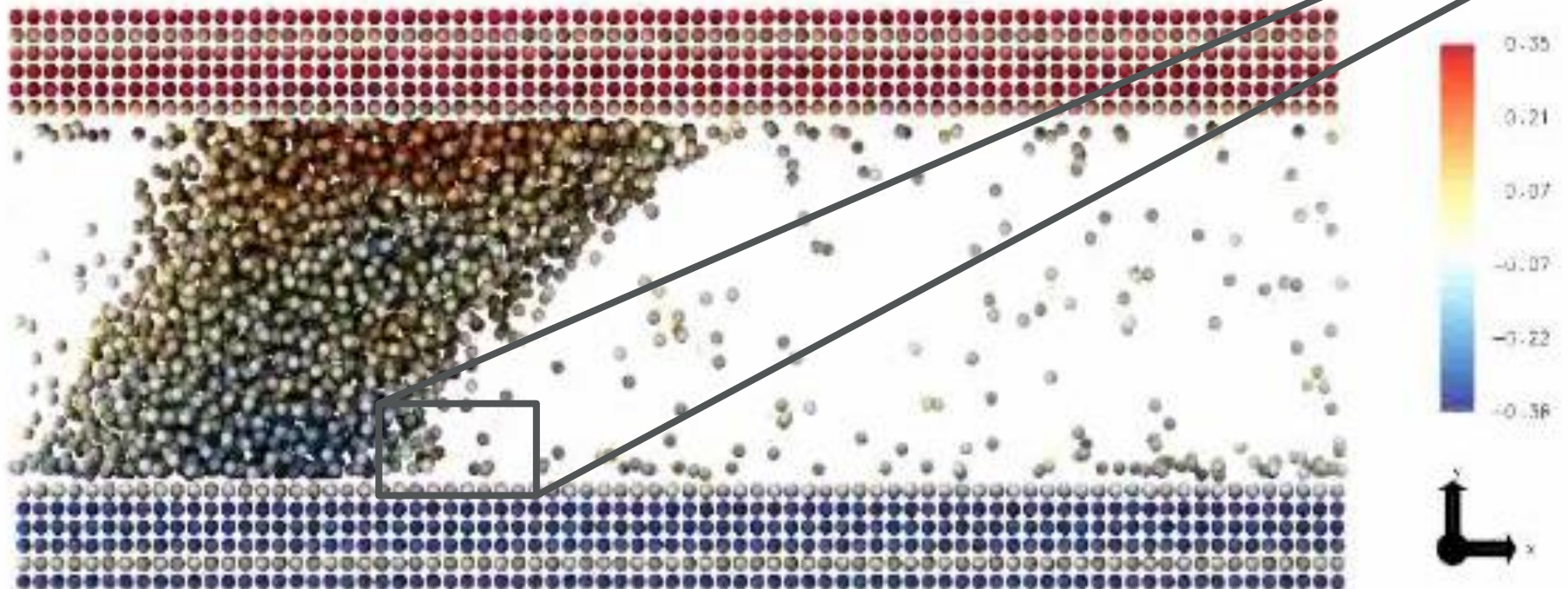
- Model the moving contact line with MD
- We want contact line speed as a function of continuum contact angle

$$\frac{dx_c}{dt}$$

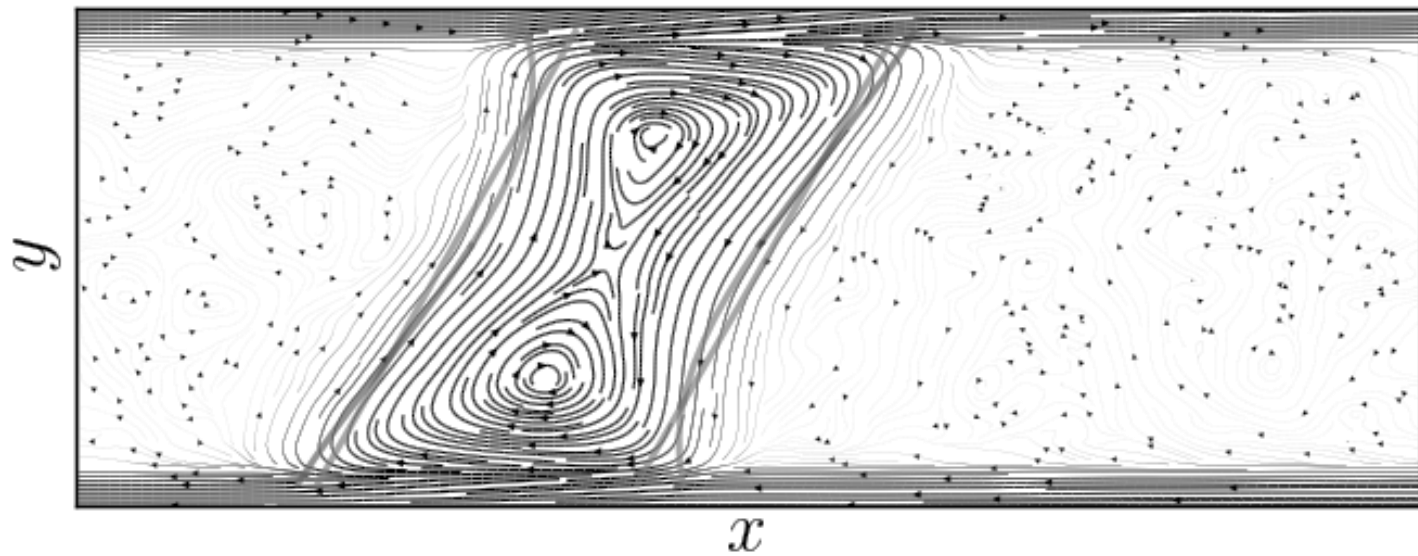
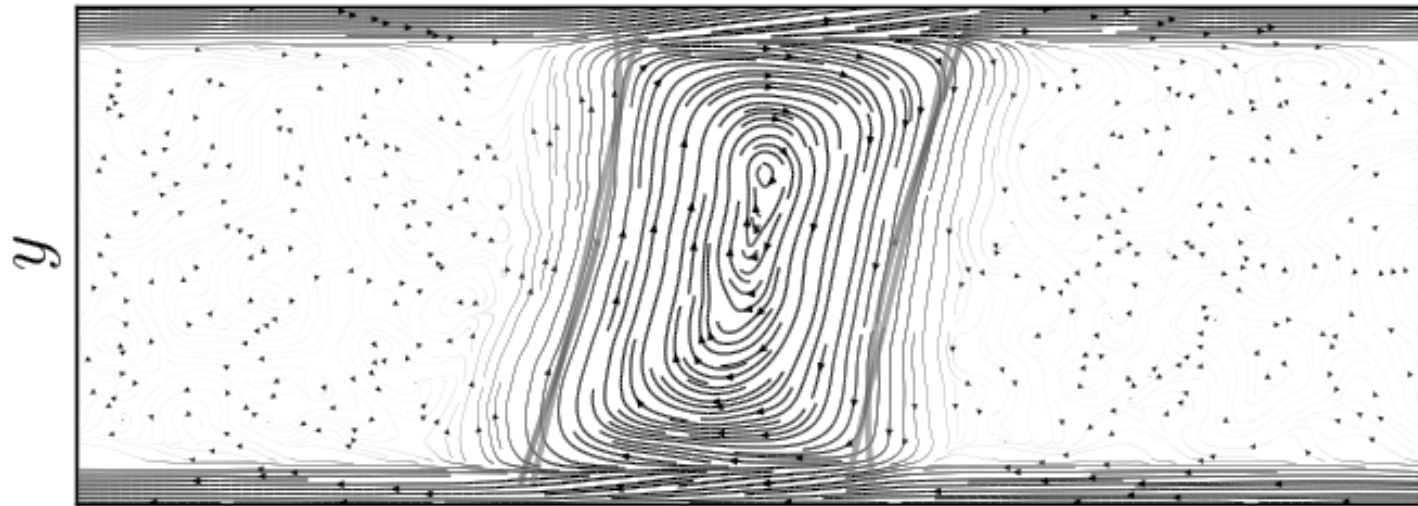


## A Sheared Liquid Bridge

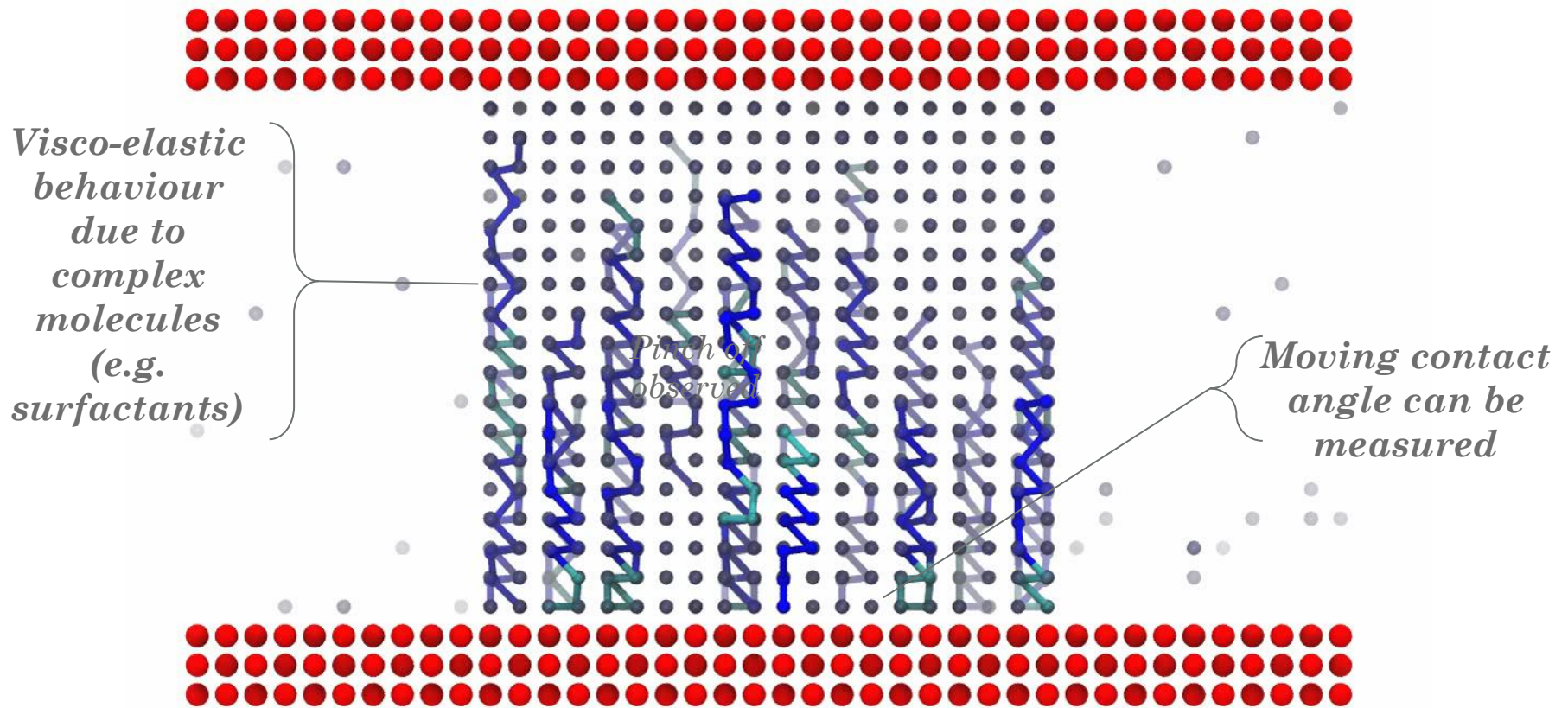
- Two fluid phases and sliding molecular walls
- Simple test case to explore wall velocity vs contact line angle
- Non-Equilibrium Steady State



# Streamlines



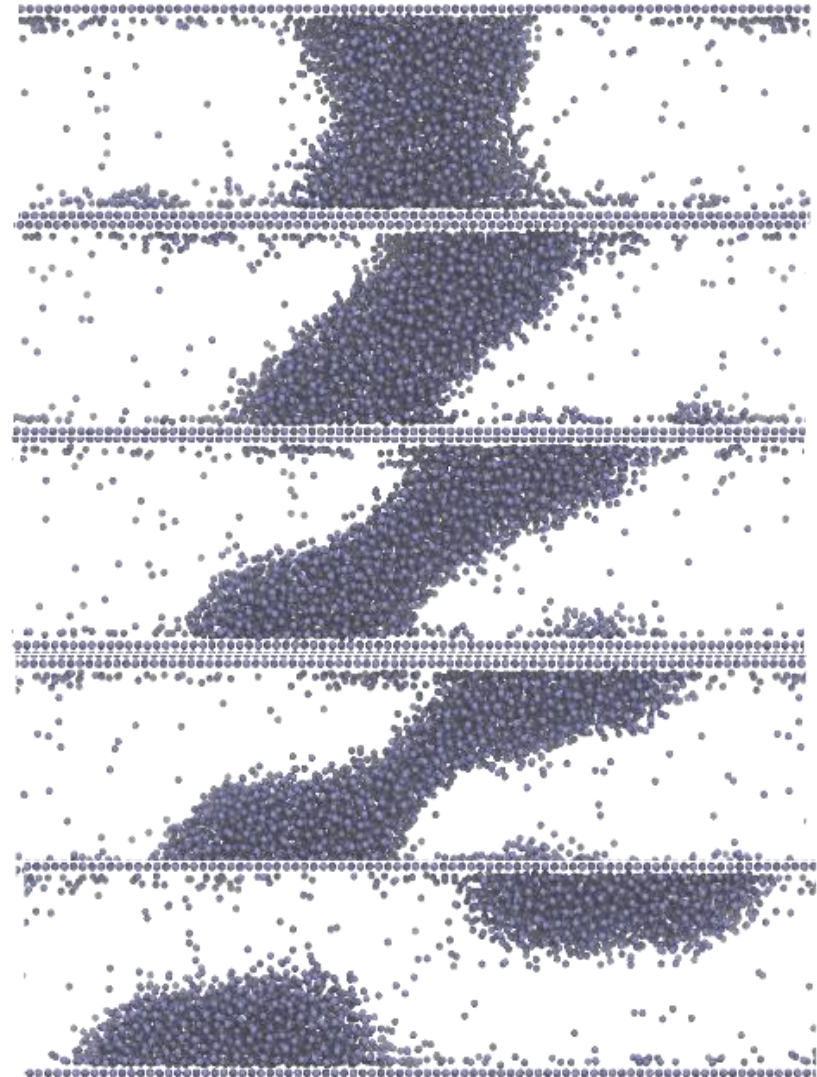
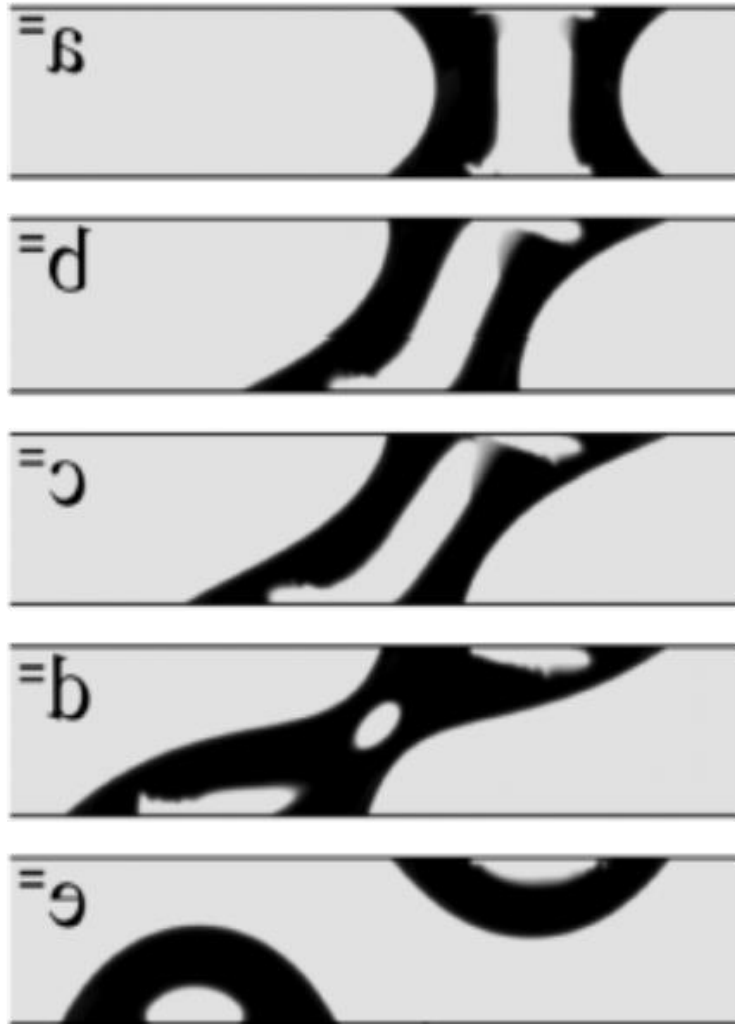
# Pinch off in a Liquid Bridge





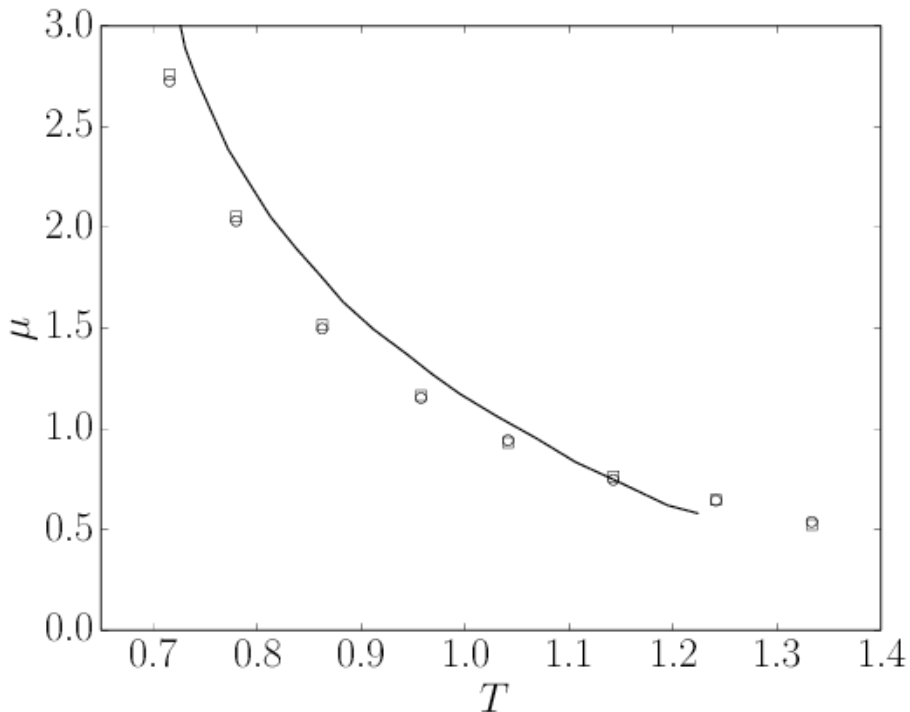
# Droplet Breakdown

L. Wang, T. J. McCarthy (2013) Shear Distortion and Failure of Capillary Bridges. Wetting Information Beyond Contact Angle Analysis Langmuir 29, 7776–7781

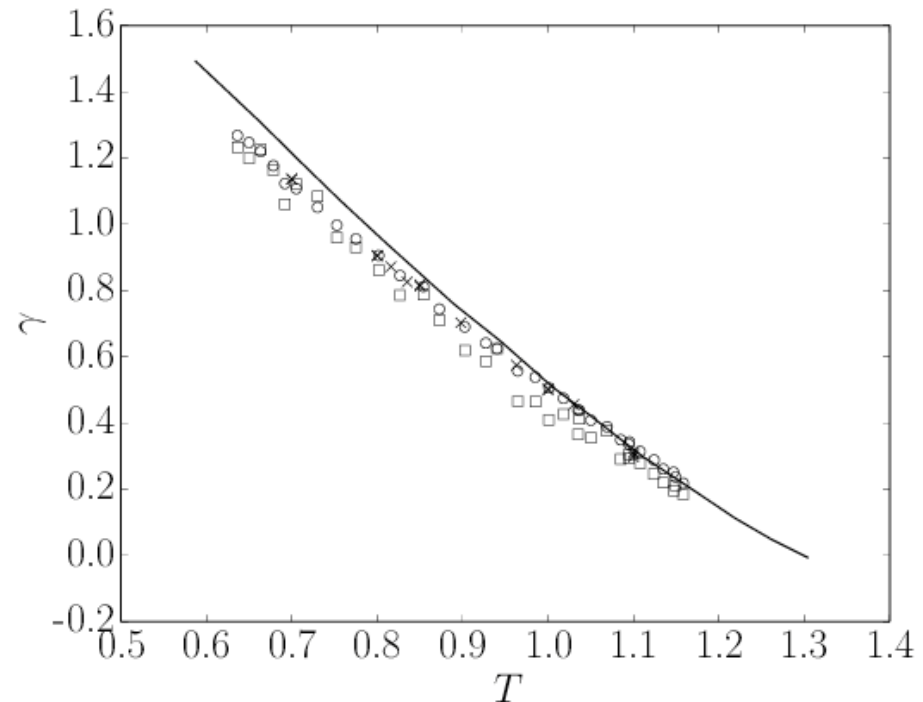


## Quantitative Validation

- Molecular Dynamics can give a-priori values of viscosity, surface tension and models the near-wall slip



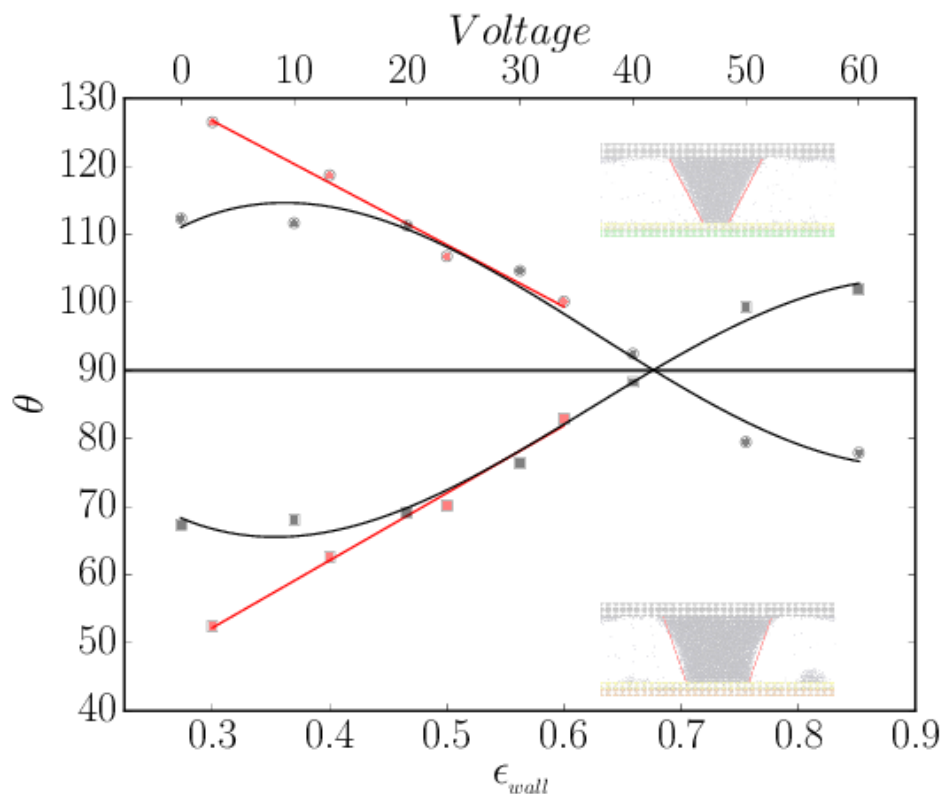
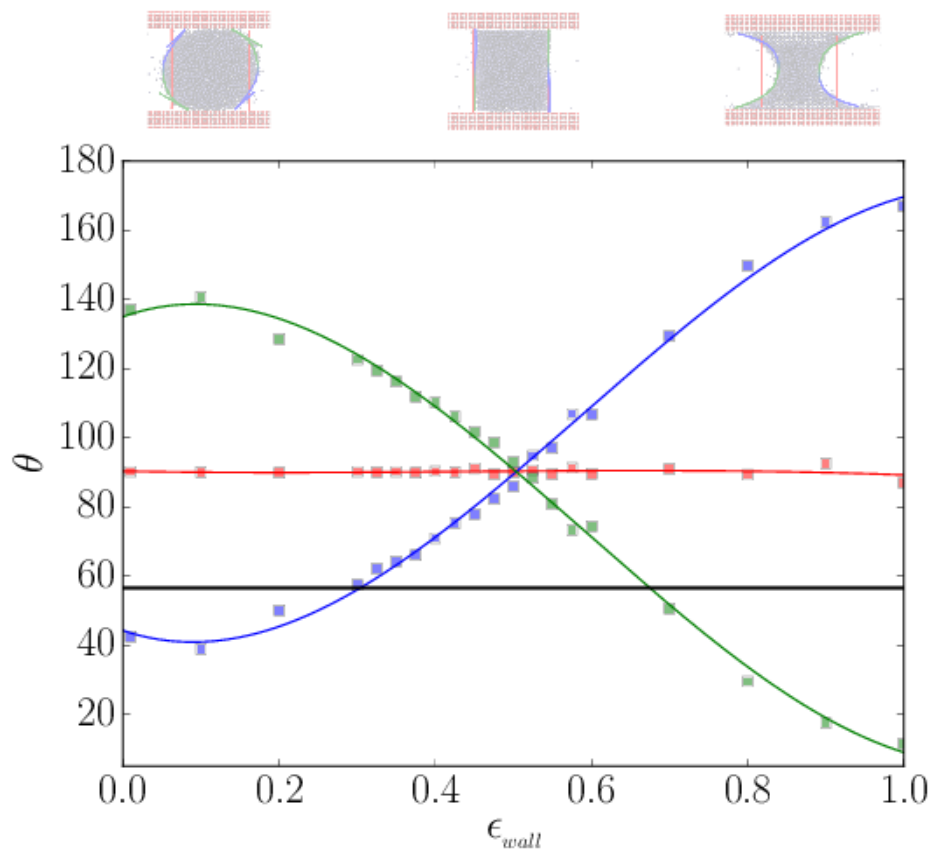
Viscosity vs Temperature



Surface Tension vs Temperature

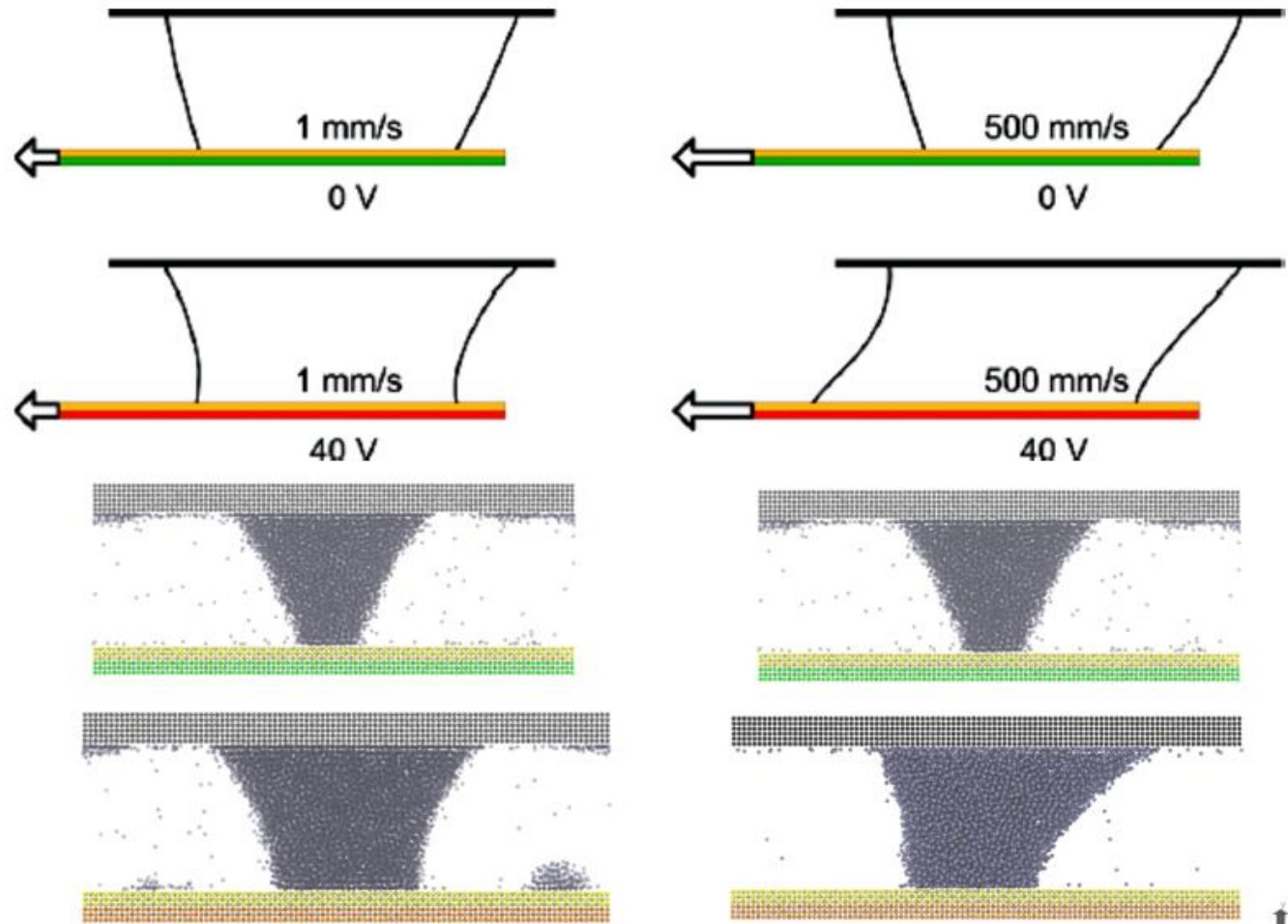
## Tuning Wall-Fluid Interaction

- A major source of MD modelling uncertainty is interaction strength between wall molecules and fluid molecules



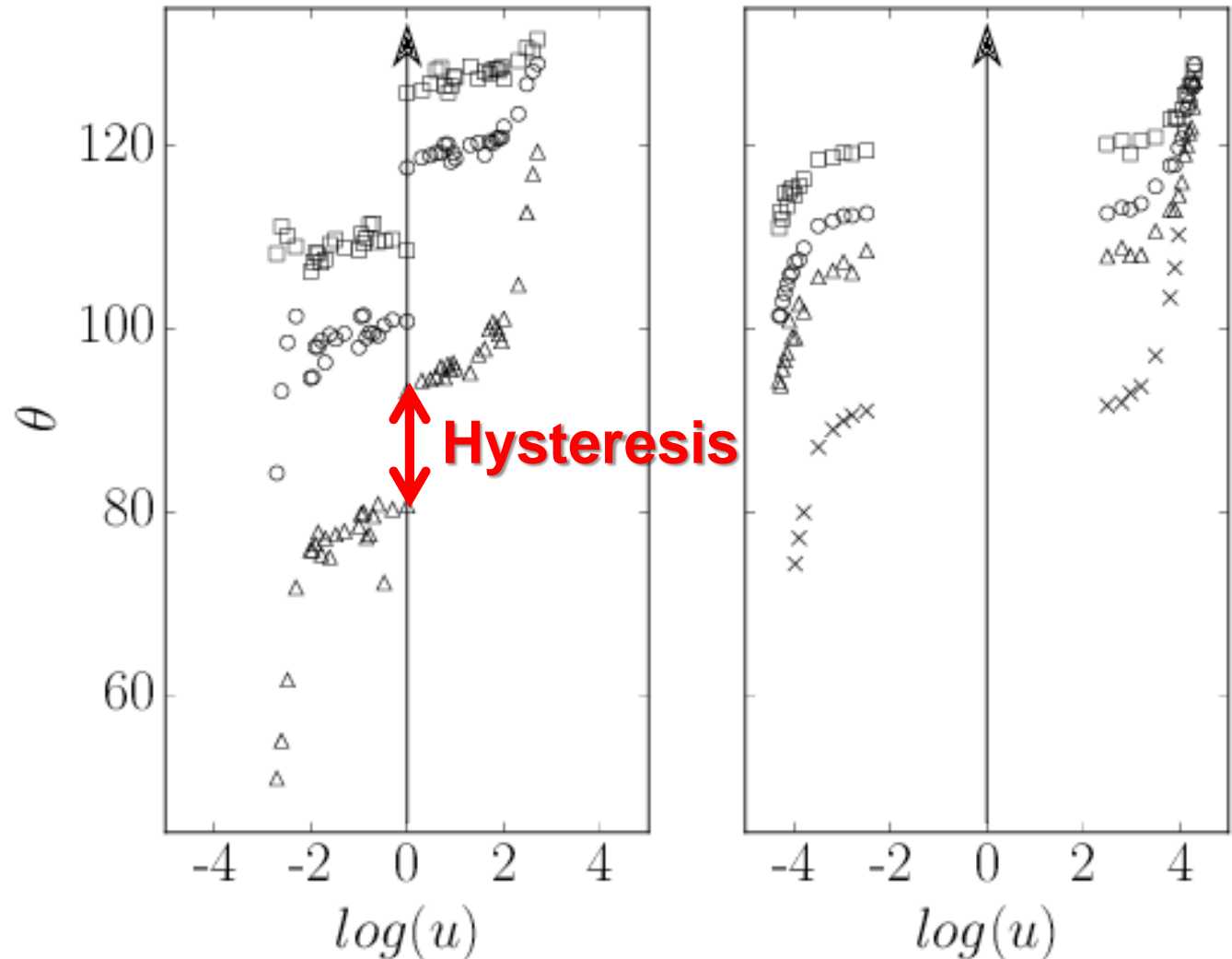
## Studying Wall Sliding with Electrowetting

- Molecular compared to Experimental work by Nelson et al (2011)
- Sliding bottom wall for range of electro-wetting



## Studying Wall Sliding with Electrowetting

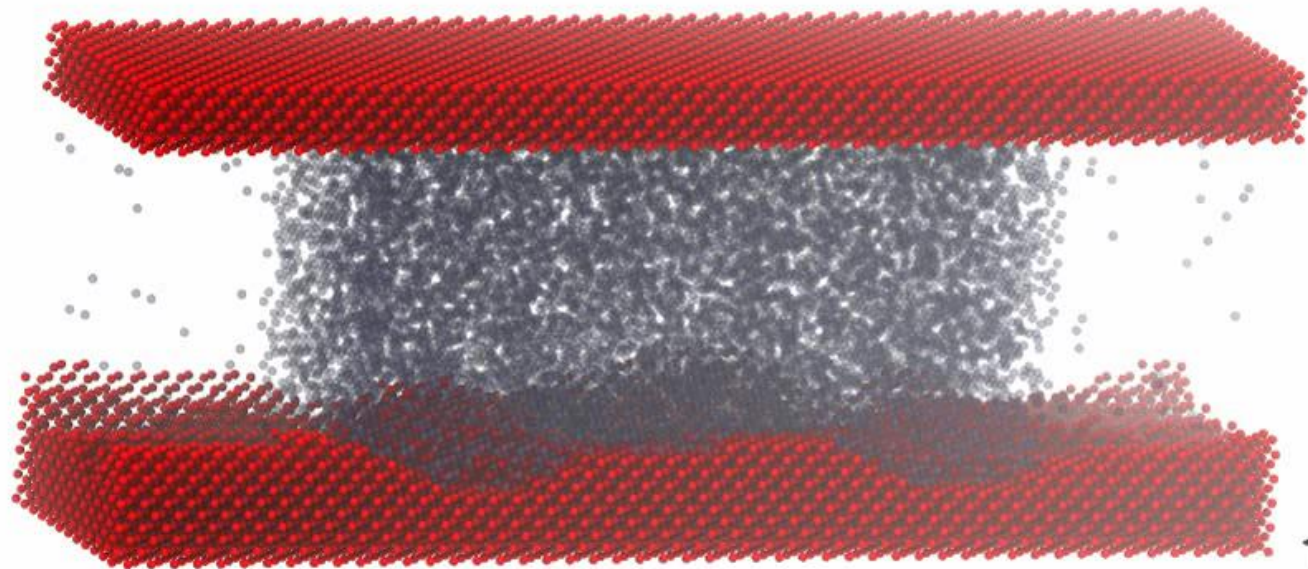
- Molecular compared to Experimental work by Nelson et al (2011)
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## Modelling Electrowetting

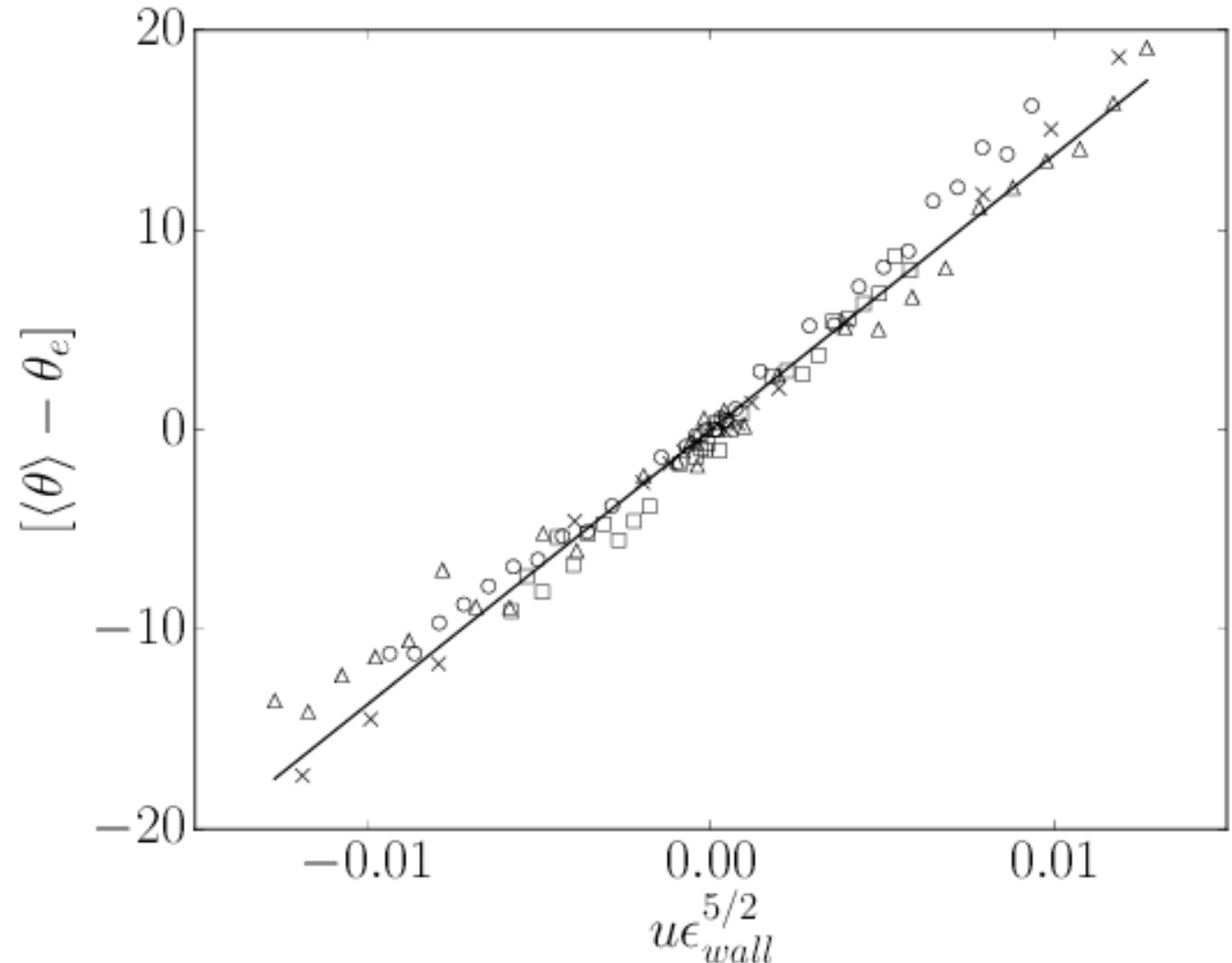
- Disagreement may be explained by
  - Lack of modelling of the rough wall
  - Over-simplified fluid model
  - Electrowetting modelled by simply adjusting interaction, electric field model could be used.
  - System size effects

**Pinning may  
result in  
hysteresis**



## How to Use Wall Sliding with Electrowetting

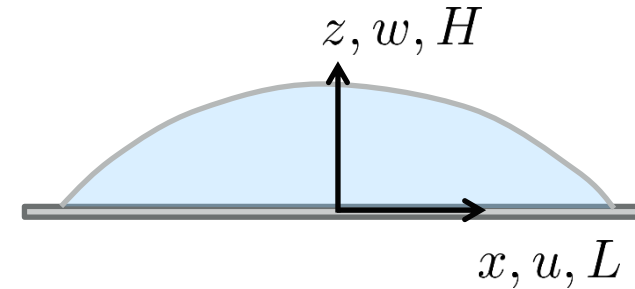
- Introduce a quantity which combines wall velocity and electrowetting to collapse data onto a single curve
- This can then be used in a continuum model



## Integrating into a CFD model

- Incompressible Navier Stokes with the thin-film approximation (see 1).

$$\frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial z^2} \quad \frac{\partial P}{\partial z} = 0 \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$



- With boundary conditions

$$P = - \left( \frac{H}{L} \right)^2 \frac{\partial^2 h}{\partial x^2} \sigma_l \quad \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = w \quad \frac{\partial u}{\partial z} = 0 \quad z = h$$

$$u = \beta \frac{\partial u}{\partial z} \quad w = 0 \quad z = 0$$

- Contact line evolution can be modelled by this reduced equation

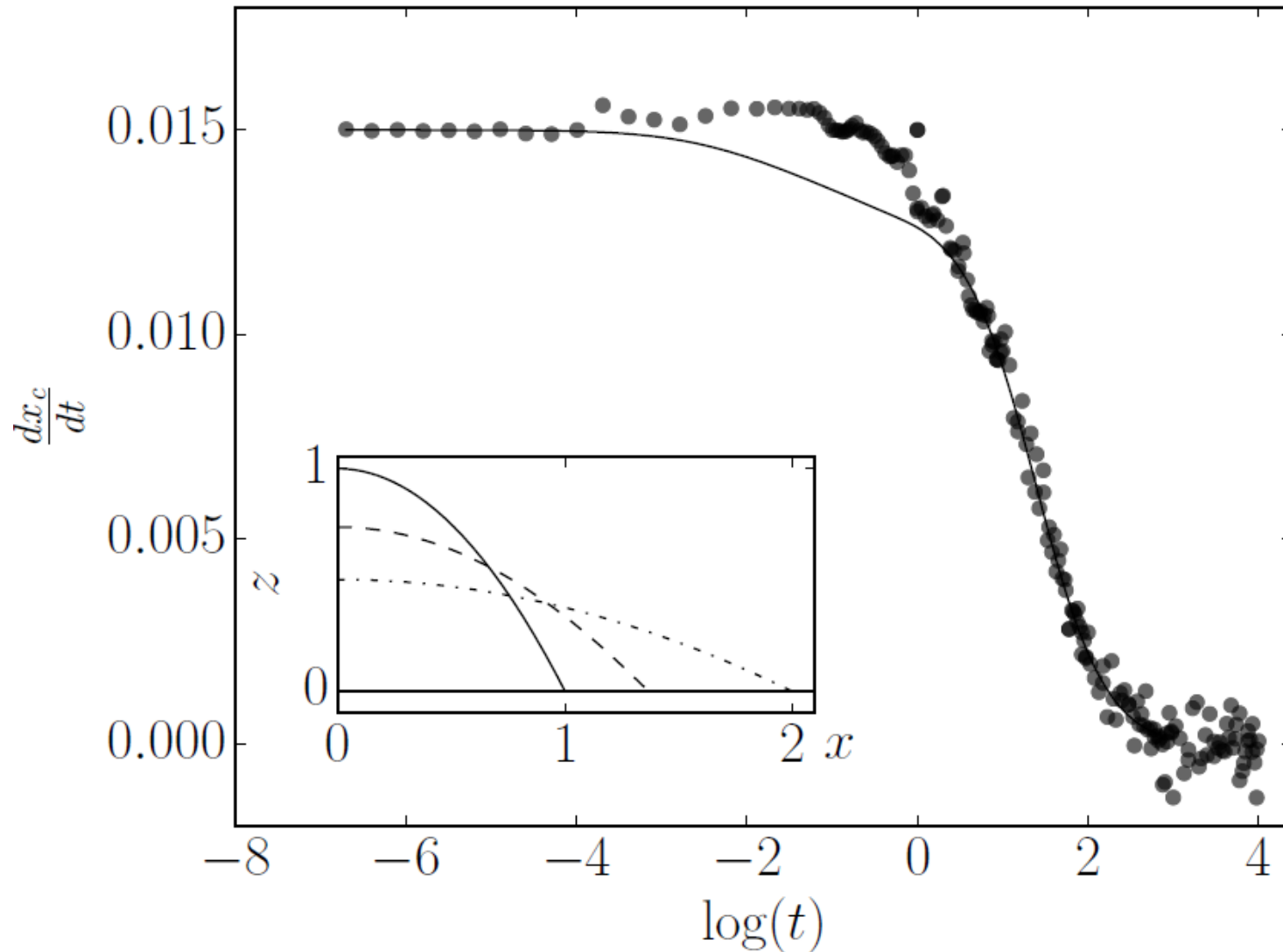
$$\frac{dx_c}{dt} = (\langle \theta \rangle - \theta_a) / E_{\text{wall}}^{5/2}$$

- Evolution of contact line can include molecular fluctuations (see 2)

$$\theta^{t+1} = \theta^t - \frac{k\Delta t}{\Gamma} [\theta^t - \langle \theta \rangle] + \xi \frac{\sqrt{C\Delta t}}{\Gamma}$$



# Molecular contact angle in continuum model



## Summary

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- Molecular Dynamics predicts a-priori viscosity, surface tension and qualitative behaviour in experiments.
- Seems ideal to model the contact line, dewetting and pinning as part of a multi-scale model (and has been used extensively in the literature)
- Choosing fluid-wall interaction is a problem
- Tuned using experiments for static contact angles, then applied to the dynamic case with poor results.
- Course grained behaviour with molecular fluctuations can be parameterised and included in a CFD model